本試題是否可以使用計算機： ☑可使用 ， □不可使用 （請命題老師勾選）

1. (20%) (a) (10%) Find e^{At} for the matrix $A=\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

(b) (5%) Consider the stable transfer function $G(s)=\dfrac{Y(s)}{U(s)}=\dfrac{s}{s^2+2s+1}$. If the steady-state response to the sinusoidal input $u(t)=\sin(5t)+2\cos(10t)$ is $y_{ss}(t)=A\sin(5t+\theta_1)+B\cos(10t+\theta_2)$, please find the values of A, B, θ_1 and θ_2.

(c) (5%) Find the Laplace transform of the periodic signal $x(t)$ as shown in Fig. 1.

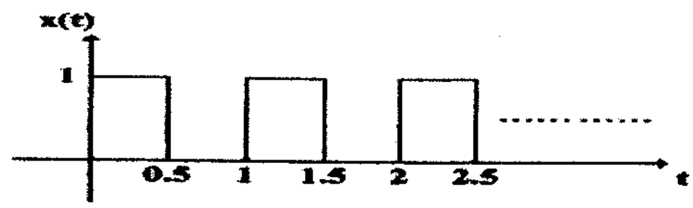

Fig. 1

2. (20%) Consider the RC-circuit shown in Fig. 2.

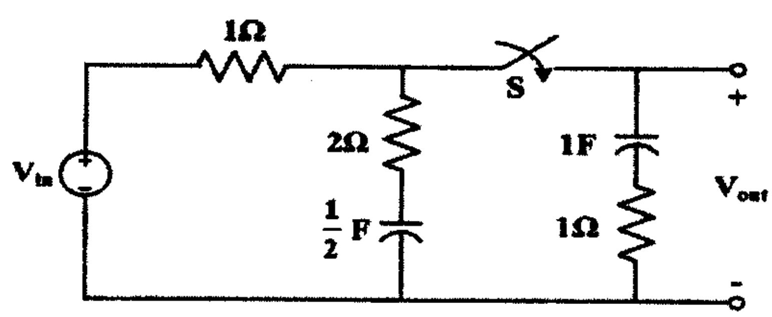

Fig. 2

(a) (5%) Please determine the transfer function from V_{in} to V_{out} when the switch S is closed.

(b) (5%) Please write down the state equation and output equation in the matrix form when the switch S is closed.

(c) (5%) Is the circuit controllable when the switch S is closed?

(d) (5%) If the input voltage $V_{in}=5$ is applied when S is set in the open position before $t=0$. At time $t=0$ the switch S is closed and the input voltage is now changed into $V_{in}=-5$. Please determine the output voltage $V_{out}(t)$ for $t\ge 0$. What is the $V_{out}(\infty)$? Verify it by the Final Value Theorem.

3. (10%) A feedback control system is shown in Fig. 3, where $G(s) = \frac{1}{s^3 + 2s^2 + 2s + 1}$ and K is the gain of proportional controller. Please use the Routh-Hurwitz criterion to determine that for the values of K the system is stable.

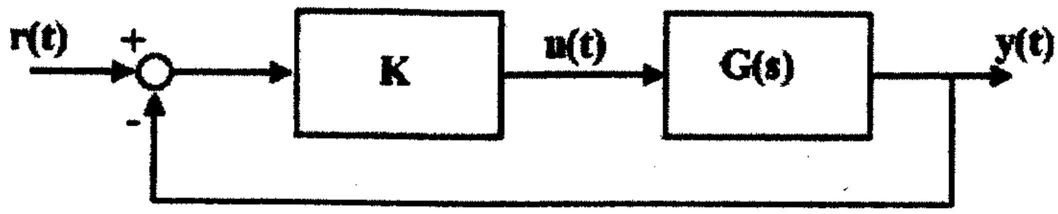

Fig. 3

4. (20%) A feedback control system is shown in Fig. 3, where $G(s) = \frac{1}{s^2 + 2s}$ and K is the gain of proportional controller.
   (a) (4%) Plot the root locus for $0 \le K < \infty$.
   (b) (4%) Determine the values of K such that the settling time $t_s \le 4$.
   (c) (4%) Determine the values of K such that the rise time $t_r \le 1$.
   (d) (4%) Determine the values of K such that the overshoot $M_p \le 0.1$.
   (e) (4%) Determine the value of K such that the closed-loop system has poles at $-1 \pm j$.

5. (15%) A feedback control system is shown in Fig. 4.
   (a) (5%) Find the transfer function relating the output $y(t)$ to the disturbance $w(t)$ when the reference input $r(t) = 0$.
   (b) (5%) Find the steady-state response $y_{ss}(t)$ if $w(t)$ is a unit ramp function.
   (c) (5%) What type of this system in relation to the reference input $r(t)$? What are the values of the step-error and ramp-error constants?

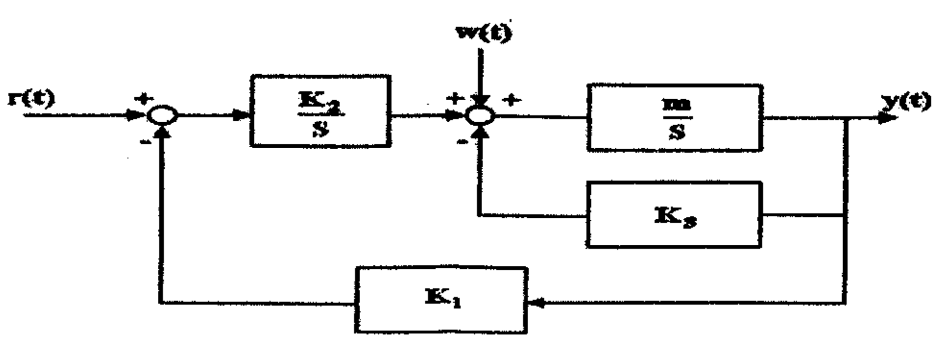

Fig. 4

本試題是否可以使用計算機： ☑可使用 ， □不可使用 （請命題老師勾選）

6. (15%) A second-order nonlinear system is described by the following state equations:

$$\dot{x} = -x + y^2$$
$$\dot{y} = x - 4$$

(a) (5%) Find the equilibrium points of this system.

(b) (5%) Find the linearized state equations about the equilibrium points.

(c) (5%) Are the lineaized systems stable?