本試題是否可以使用計算機：☐可使用，☑不可使用　（請命題老師勾選）

1. (a) Using Lagrange interpolation to find $f(x)$ (10%) and calculate (b) $\int_{x_1}^{x_3} f(x)dx$ for the data $\{x_i, f(x_i)\}_{i=1}^{i=3}$ with $x_i = (i-1)h + x_1$. (5%)

   (c) Using the above result to obtain the expression of $\int_{x_1}^{x_7} f(x)dx$ in terms of h, x_i and $f(x_i)$ for $i = 1 \sim 7$. (10%)

2. Suppose p to be one solution of $f(x) = 0$ obtained by Newton' method

   $$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

   Questions: (a) Plot the meaning of the iteration (10%), and (b) find the rate of convergence (15%).

3. The Runge-Kutta method for solving $y' = f(t, y)$ with the initial conditions $y(t_0) = \alpha$ is expressed as

   $$w_0 = \alpha,\ k_1 = hf(t_i, w_i),\ k_2 = hf(t_i + \frac{1}{2}h, w_i + \frac{1}{2}k_1),\ k_3 = hf(t_i + \frac{1}{2}h, w_i + \frac{1}{2}k_2),$$

   $$k_4 = hf(t_{i+1}, w_i + k_3),\ w_{i+1} = w_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4),\ h = t_{i+1} - t_i,$$

   for $i = 0, 1, \ldots$.

   Question: How to use the Runge-Kutta method to solve $y'' = f(t, y, y')$ with $y(t_0) = \alpha_1$ and $y'(t_0) = \alpha_2$ (25%).

4. (Least square method) The $y = ax + b$ is used to approximate the data $\{x_i, f(x_i)\}_{i=1}^{i=n}$. Question: Find the value of $\sum_{i=1}^{n} [y_i - f(x_i)]^2$. (25%)