

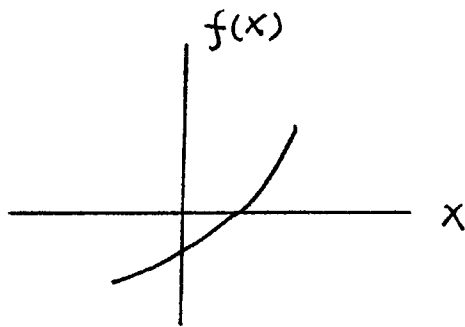
編號: 150 系所: 工程科學系乙組

科目: 數值分析

本試題是否可以使用計算機: 可使用, 不可使用 (請命題老師勾選)

1. Suppose that the equation $f(x) = 0$ has one real root. The following two methods are adopted to solve $f(x) = 0$. Questions: Describe the steps of solving $f(x) = 0$ and plot the corresponding figures.

- (a) Bisection method (10%)
 (b) Newton's method (15%).



2. Usually, a function $f(x)$, $x \in [a, b]$, can be approximated as a polynomial by dividing the domain length into $2n$ equal intervals. Then Lagrange's interpolation is taken to finish the work. Questions:

- (a) Write down the polynomial created by Lagrange's interpolation. (8%)
 (c) Adopt the result of (a) to write the polynomial created by $\{x_i, f(x_i)\}_{i=0}^{i=2n}$ and

calculate $\int_{x_0}^{x_2} f(x) dx$. (8%)

- (d) Adopt the result of (b) to calculate $\int_{x_0}^{x_{2n}} f(x) dx$. (9%)

3. The following two methods are taken to solve the set of equations $f_i(x_1, \dots, x_n) = 0$, $i = 1 \sim n$. Questions: Describe the main concept of each method and write down the approach.

- (a) Newton's method. (10%)
 (b) Steepest gradient method. (15%)

4. The following two finite-difference methods are taken to solve the problem

$$\frac{d^2 y}{dx^2} + g(x) \frac{dy}{dx} + k(x)y = f(x), \quad x \in [a, b], \quad y(a) = \alpha, \quad y(b) = \beta.$$

Set $x_i = x_0 + ih$, $x_0 = a$, $i = 1 \sim n+1$, $x_{n+1} = b$, $h = (b-a)/(n+1)$.

Questions: Write down the vector form of results obtained by each method:

- (a) Forward-difference method. (12%)
 (b) Backward-difference method. (13%)