1. Solve the partial-differential equation

$$\alpha \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}, \;\; 0 \le x \le L, \;\; 0 \le t,$$

with the conditions $T(0,t)=0$, $T(L,t)=f(t)$, $T(x,0)=g(x)$. (25%)

2. Calculate $\displaystyle\int_0^\infty \frac{\cos(mx)}{(x^2+a^2)(x^2+b^2)} dx$　(25%)

3. Find the solution of $\;x^3 \dfrac{d^3 y}{dx^3} - 8x^2 \dfrac{d^2 y}{dx^2} + 55x \dfrac{dy}{dx} - 123y = x^3;\;$ (25%)

4. Solve $X^2 - 5X + 3I = \begin{bmatrix} 1 & -4 \\ 2 & -5 \end{bmatrix}$.　(25%)