

本試題是否可以使用計算機： 可使用， 不可使用（請命題老師勾選）

考試日期：0301，節次：1

1. The following partial differential equation with the boundary conditions

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y), \quad 0 \leq x, y \leq 3,$$

$$u(x, 0) = g_1(x), \quad u(x, 3) = g_2(x), \quad u(0, y) = h_1(y), \quad u(3, y) = h_2(y),$$

can be expressed as the following matrix form by the finite-difference method

$$A\bar{U} = \bar{F},$$

where $x_0 = y_0 = 0$, $x_1 = y_1 = 1$, $x_2 = y_2 = 2$, $x_3 = y_3 = 3$, and

$$\bar{U} = \{u(1,1), u(2,1), u(1,2), u(2,2)\}^T.$$

Questions: What are the matrix A and the vector \bar{F} ? (30%)

2. The Gaussian quadrature for the following integral is expressed as

$$\int_{-1}^1 f(x) dx = \sum_{i=1}^n c_i f(x_i)$$

Questions: Using the Gaussian quadrature to calculate the following two integrals

(a) $\int_{-1}^1 f(x) dx$, (10%) and (b) $\iint f(x, y) dy dx$, $0 \leq x^2 + y^2 \leq a^2$. (30%)

3. The Runge-Kutta method of order 4 for the differential equation

$$y' = f(t, y), \quad a \leq t \leq b, \quad y(a) = \alpha$$

is given by

$$w_0 = \alpha, \quad k_1 = hf(t_i, w_i), \quad k_2 = hf\left(t_i + \frac{1}{2}h, w_i + \frac{1}{2}k_1\right), \quad k_3 = hf\left(t_i + \frac{1}{2}h, w_i + \frac{1}{2}k_2\right),$$

$$k_4 = hf(t_{i+1}, w_i + k_3), \quad w_{i+1} = w_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4).$$

Question: Extended the Runge-Kutta method for the following equations

$$u_1' = f_1(t, u_1, \dots, u_n), \dots, u_n' = f_n(t, u_1, \dots, u_n) \quad \text{with}$$

$$u_1(a) = \alpha_1, \dots, u_n(a) = \alpha_n. \quad (30\%)$$