※ 考生請注意：本試題 □可　☑不可 使用計算機

1. Solve the equation $\nabla^2 u = f(x,y)$, $0 \le x \le a$, $0 \le y \le b$ with the boundary conditions $\dfrac{\partial u}{\partial y}(x,0) = \dfrac{\partial u}{\partial y}(x,b) = 0$, $u(0,y) = 0$, $u(a,y) = 0$. (25%)
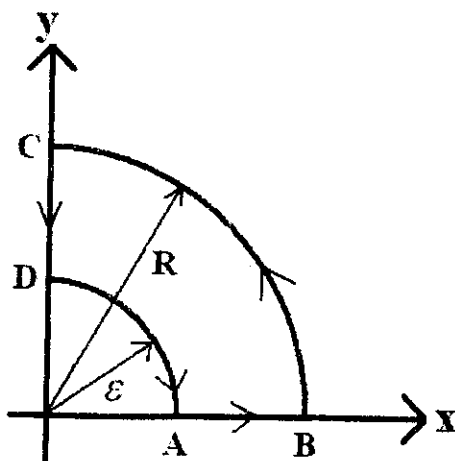
2. a. A surface is described by the function $z = f(x,y)$, $0 \le x \le a$, $0 \le y \le b$

   Find the area σ of the surface. (10%)

   b. Evaluate $\displaystyle\iint\limits_{\Sigma} z \, d\sigma$ with Σ that part of the plane $x + y + z = 4$ lying above the rectangle $0 \le x \le 2$, $0 \le y \le 1$. (15%)

3. Solve $\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2} + f(x,t)$, $t > 0$, $0 \le x \le L$, with the conditions

   $u(0,t) = u(L,t) = 0$, $u(x,0) = p(x)$, and $\dfrac{\partial u}{\partial t}(x,0) = 0$. (25%)

4. Show that $\displaystyle\int_0^\infty \frac{\cos x}{\sqrt{x}}dx = \int_0^\infty \frac{\sin x}{\sqrt{x}}dx = \sqrt{\frac{\pi}{2}}$ by integrating $\dfrac{e^{iz}}{\sqrt{z}}dz$ around the

   contour. (25%)

$R \to \infty$, $\varepsilon \to 0$