系所組別： 工程科學系乙組

考試科目： 數值分析

※ 考生請注意：本試題 □可 ☑不可 使用計算機

1. 20%

To find roots for a **nonlinear** function $f(x)=0$, we often use Newton Iteration method. Please derive the relation used in the method and what is its convergent rate?

2. 20%

The least-square method can be used to find an polynomial function to approxmiate a set of discrete data. Please explain and derive how to find this function ?

3. 20%

(a) Lagrangian Polynomials can be used to interpolate a set of data points (x_i, f_i), $i=0,1,2,\ldots\ldots n$. Lets write the polynomial as $P_n(x)=\sum_{i=0}^{n} l_i(x) f_i$.

Find $l_i(x)=?$ (5%) Analyze the error caused by the interpolation. (5%).

(b) We can use divided difference to obtain interpolation polynomial as:

$P_n(x)=a_0+a_1(x-x_0)+a_2(x-x_0)(x-x_1)+\ldots\ldots+a_n(x-x_0)(x-x_1)\ldots(x-x_{n-1})$,

Find $a_i=?$ i=1,2....n. (5%)

(c) We can use either Lagrangian polynomial or divided difference in polynomial interpolation. Would the two resulting polynomials in (a) and (b) be different? Why? Explain your answer.(5%).

4. (20%) Derive the following finite difference formule:

(a) $\dfrac{\partial f_i}{\partial x}=\dfrac{f_{i+1}+(\sigma^2-1)f_i-\sigma^2 f_{i-1}}{\sigma(\sigma+1)\Delta x}+O(?a)$, where $\sigma=\dfrac{x_{i+1}-x_i}{x_i-x_{i-1}}, \Delta x=x_i-x_{i-1}$, (5%)

(b) $\dfrac{\partial^2 f_i}{\partial x^2}=2\dfrac{f_{i+1}-(1+\sigma)f_i+\sigma f_{i-1}}{\sigma(\sigma+1)\Delta x^2}+O(?b)$ (5%)

(c) $\dfrac{\partial f_i}{\partial x}=\dfrac{Af_i+Bf_{i-1}+Cf_{i-2}}{D\Delta x}+O(?c)$, where $\lambda=\dfrac{x_i-x_{i-1}}{x_{i-1}-x_{i-2}}, \Delta x=x_i-x_{i-1}$ What are the truncation errors of (a), (b) and (c)? What are A, B, C, and D? (5%)

(d) $\dfrac{\partial f_i}{\partial x}=\dfrac{2f_{i+3}-9f_{i+2}+18f_{i+1}-11f_i}{6\Delta x}+O(\Delta x)^3$ for equal spacing mesh Δx . (5%)

5. 20%

Briefly describe and write down the mathematical expressions for the following numerical methods in solving a system of equations Ax=b, A is a n x n matrix, x and b are n x 1 column vectors.

(a) The Gaussian elimination method (5%)
(b) The Gauss-Seidel iteration method (5%)
(c) The Jacobi iteration method (5%)
(d) The successive over relaxation method (5%)