※ 考生請注意：本試題 □可 ☑不可 使用計算機

1. 30%

We want to use the Newton's iteration method to solve the following nonlinear equations:

$$\begin{cases} x_1^2 + x_2 - 37 = 0 \\ x_1 - x_2^2 - 5 = 0 \\ x_1 + x_2 + x_3 - 3 = 0 \end{cases}$$

Derive the iteration expression for finding the roots.

2. 20%

The least-square method can be used to find an polynomial function of degree n to approxmiate a set of m pairs discrete data. Explain how to find this function ?

3. 30%

Gaussian Quadrature　　　$\int_{-1}^{1} f(t)dt = \sum_{i=1}^{n} w_i f(t_i)$

The Legendre polynomials are defined by recursion:

$(n+1)L_{n+1}(x) - (2n+1)xL_n(x) + nL_{n-1}(x) = 0$　with　$L_0(x) = 1$,　$L_1(x) = x$

(a) Find $L_3(x) = ?$ (10%)

(b) Find the three-term *Gaussian* quadrature nodes, (ie t values). (10%)

(c) Find the weighting factors for the corresponding nodes, (ie w values). (10%)

4. 20%

Consider a 1-D partial differential equation　$\dfrac{\partial T}{\partial t} = \alpha \dfrac{\partial^2 T}{\partial x^2}$,　$\alpha = $ constant > 0, with the following finite difference scheme:

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = \alpha \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{(\Delta x)^2}$$

What is the stability condition of the method?