1. Solve the initial value problem

$x^2y' + 2xy - x + 1 = 0, \quad y(1) = 0.$ (10%)

2. Solve

$x^2y'' - 3xy' + 4y = 4.$ (10%)

3. Find the Laplace transform of the function

$f(t) = t\cos(\beta t) + \frac{1}{\beta}\sin(\beta t).$ (10%)

4. Find the eigenvectors of the matrix

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}.$$ (10%)

5. Find the directional derivative $\partial f/\partial s$ of $f(x,y,z) = 2x^2 + 3y^2 + z^2$ at the point P:(2,1,3) in the direction of the vector $\vec{a} = \vec{i} - 2\vec{k}$. (10%)

6. Evaluate the following integral on any path C from P:(0,0,1) to Q:(1,π/4,2).

$$I = \int_C \{ 2xyz^2\,dx + [\, x^2z^2 + z\cos(yz)\,]\,dy + [\, 2x^2yz + y\cos(yz)\,]\,dz \}.$$ (10%)

7. The differential equation

$y'' + \frac{a(x)}{x}y' + \frac{b(x)}{x^2}y = 0, \qquad (x > 0),$

has a series solution of the form

$y_1(x) = x^r\,(c_0 + c_1x + c_2x^2 + \cdots),$

where $c_0 \neq 0$ and r is a double root of the indicial equation. Show that a second linearly independent solution of the differential equation is of the form

$$y_2(x) = y_1(x)\ln(x) + x^r\sum_{m=0}^{\infty} A_m x^m$$ (20%)

8. Solve the problem

$\frac{\partial u}{\partial t} = a^2\frac{\partial^2 u}{\partial x^2} \qquad (-\infty < x < \infty,\ t > 0),$

$u(x,0) = e^{-|x|}, \qquad (-\infty < x < \infty).$ (20%)