

1. Verify that  $y = e^x$  satisfies the homogeneous equation associated with  $(x - 1)y'' - xy' + y = 1$ , and obtain the general solution (15%).

2. Solve (15%)

$$\begin{cases} \frac{d^2x}{dt^2} + x + 2y = 7e^{2t} - 1 \\ \frac{d^2x}{dt^2} + 3x + 2y = 9e^{2t} + 1 \end{cases}$$

3. Find the Laplace transformation of the function  $e^{-3t}f(t)$ , where (15%)

$$f(t) = \begin{cases} 0, & t < 8 \\ t^2 - 4, & t \geq 8 \end{cases}$$

4. Solve  $x^2y'' + x\left(\frac{1}{2} + 2x\right)y' + \left(x - \frac{1}{2}\right)y = 0$  ( $x > 0$ )

by the method of Frobenius.(15%).

5. Expand the function  $f(x) = 1$  in a series of the characteristic functions of the boundary - value problem

$$\frac{d^2y}{dx^2} + \lambda y = 0$$

$$y(0) = 0, \quad \alpha Ly'(L) + y(L) = 0 \quad (\alpha \geq 0)$$

over the interval  $(0, L)$  (15%).

6. Find the angle between the planes  $2x - y + 2z = 1$  and  $x - y = 2$ . (10%)

7. Given  $A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$

Find the eigenvalues  $\lambda$ 's and corresponding normalized eigenvectors  $v$  of  $A$ . How do you check your results? Find the transformation which diagonalizes  $A$  and verify your results. Is  $A$  positive definite? (15%)