系所組別： 化學工程學系甲組
考試科目： 單元操作與輸送現象
考試日期：0225，節次：1

※ 考生請注意：本試題可使用計算機，並限「考選部核定之國家考試電子計算器」機型

Equations:

The Navier-Stokes equation for an incompressible flow: $\rho\frac{D\vec{\upsilon}}{Dt}=\rho\vec{g}-\nabla P+\mu\nabla^2\vec{\upsilon}$

For **CYLINDRICAL COORDINATES**

r direction

$$\rho\left(\frac{\partial\upsilon_r}{\partial t}+\upsilon_r\frac{\partial\upsilon_r}{\partial r}+\frac{\upsilon_\theta}{r}\frac{\partial\upsilon_r}{\partial\theta}-\frac{\upsilon_\theta^2}{r}+\upsilon_z\frac{\partial\upsilon_r}{\partial z}\right)=\rho g_r-\frac{\partial P}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\upsilon_r)\right)+\frac{1}{r^2}\frac{\partial^2\upsilon_r}{\partial\theta^2}-\frac{2}{r^2}\frac{\partial\upsilon_\theta}{\partial\theta}+\frac{\partial^2\upsilon_r}{\partial z^2}\right]$$

θ direction

$$\rho\left(\frac{\partial\upsilon_\theta}{\partial t}+\upsilon_r\frac{\partial\upsilon_\theta}{\partial r}+\frac{\upsilon_\theta}{r}\frac{\partial\upsilon_\theta}{\partial\theta}+\frac{\upsilon_r\upsilon_\theta}{r}+\upsilon_z\frac{\partial\upsilon_\theta}{\partial z}\right)=\rho g_\theta-\frac{1}{r}\frac{\partial P}{\partial\theta}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\upsilon_\theta)\right)+\frac{1}{r^2}\frac{\partial^2\upsilon_\theta}{\partial\theta^2}+\frac{2}{r^2}\frac{\partial\upsilon_r}{\partial\theta}+\frac{\partial^2\upsilon_\theta}{\partial z^2}\right]$$

z direction

$$\rho\left(\frac{\partial\upsilon_z}{\partial t}+\upsilon_r\frac{\partial\upsilon_z}{\partial r}+\frac{\upsilon_\theta}{r}\frac{\partial\upsilon_z}{\partial\theta}+\upsilon_z\frac{\partial\upsilon_z}{\partial z}\right)=\rho g_z-\frac{1}{r}\frac{\partial P}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\upsilon_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2\upsilon_z}{\partial\theta^2}+\frac{\partial^2\upsilon_z}{\partial z^2}\right]$$

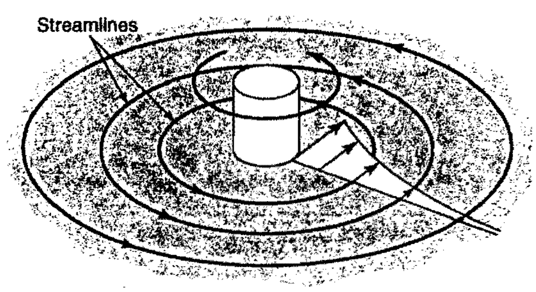
The vorticity

$$\nabla\times\vec{\upsilon}=\left(\left(\frac{1}{r}\frac{\partial v_z}{\partial\theta}-\frac{\partial v_\theta}{\partial z}\right)\vec{e}_r+\left(\frac{\partial v_r}{\partial z}-\frac{\partial v_z}{\partial r}\right)\vec{e}_\theta+\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)-\frac{1}{r}\frac{\partial v_r}{\partial\theta}\right)\vec{e}_z\right),$$

Problems (請依題號順序，依序作答)

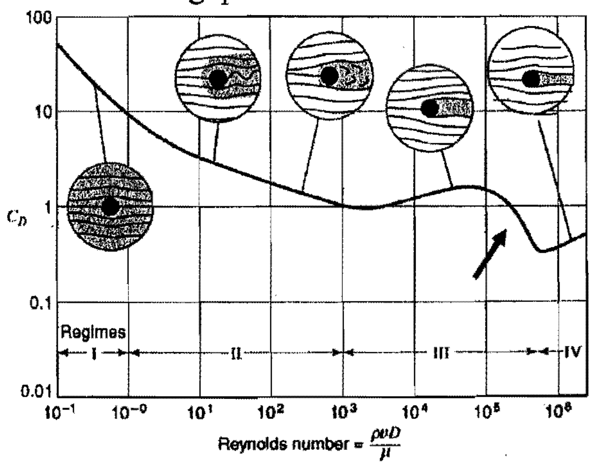
1. One graduate student and his advisor setup a home-made instrument that can measure the deformation rate of a polymer solution by applying a known shear force. Then the shear stress as a function the deformation rate for the polymer solution can be obtained. After several months of hard work, the student summarized his following findings.
   (i) As the concentration of the polymer solution is much lower than a certain concentration (c*), the applied shear stress was found to be proportional to the deformation rate of the polymer solution.
   (ii) As the concentration is higher than a certain concentration (c*), the applied shear stress was not proportional to the deformation rate.
   Please explain the observation. (4%)

2. A rotating shaft, as shown in the figure below, causes the incompressible fluid to move in circular streamlines with the velocity $\vec{\upsilon}=A\vec{e}_\theta/r$.
   (1) Prove that the pressure gradient can be expressed as $\nabla P=\rho\vec{g}-\rho\left(\frac{\partial\vec{\upsilon}}{\partial t}+\vec{\upsilon}\bullet\nabla\vec{\upsilon}\right)$. (6%)
   (2) Prove that the flow is irrotational. (5%)



※ 考生請注意：本試題可使用計算機，並限「考選部核定之國家考試電子計算器」機型

3. The drag coefficient (C_D) for circular cylinders moving through a viscous fluid as a function of Reynolds number (Re) is shown below. Please answer the following questions.
   (a) Please specify the predominating forces in regime I and IV, respectively. (3%)
   (b) Please explain the marked decrease in drag observed in regime III (indicated by the arrow). (3%)
   (c) Can the equation, $\nabla^2\Psi = 0$, be the governing equation to derive the fluid flow in regime I? Ψ is the stream function. Please explain. (3%)



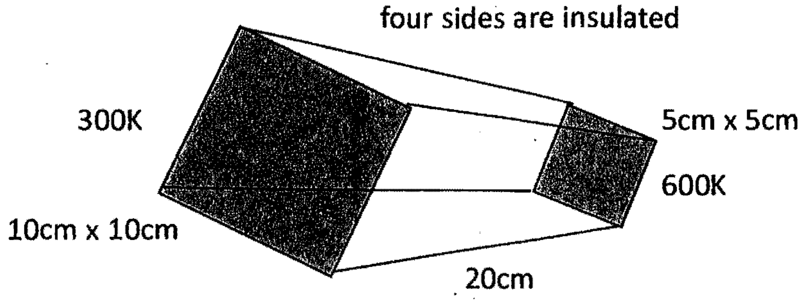
4. The head loss for a given fitting can be evaluated as: $h_L = K\frac{\upsilon_{avg}^2}{2g}$ where K is the friction loss factor.
   (a) What would influence the parameter K for a given fitting? (3%)
   (b) For the following fittings, globe valve, 180° bend, and 45° elbow, which one would have the highest friction loss factor? Please explain. (3%)

5. Consider a cylindrical solid with internal energy generation. Assume only radial conduction occurs:
   (a). derive the following differential equation: (8%)

$$\dot{q} + \frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \rho c_p \frac{\partial T}{\partial t}$$

   where $\dot{q}$ is the rate of energy generated per unit volume, **k**: thermal conductivity, c_p :heat capacity, ρ : density, T: temperature, **t**: time
   (b). what is the energy flux in radial direction? (7%)

6. The following object has a small end (5 cm x 5 cm) and a large end (10cm x 10cm) and its height is 20 cm. The temperature for the small end and large end is held at 600K and 300K, respectively. What heat-flow rate will be obtained if the four sides are insulated? Assume one dimensional heat conduction and the thermal conductivity is 0.2 W/(m · k) (10%)

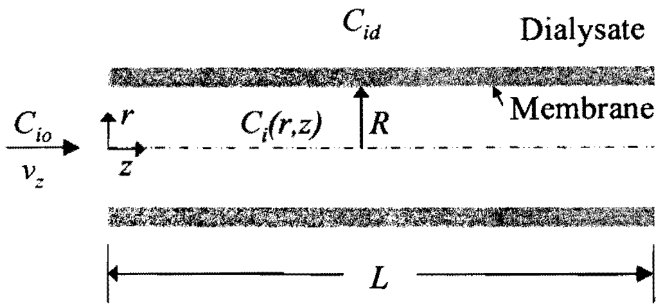


※ 考生請注意：本試題可使用計算機，並限「考選部核定之國家考試電子計算器」機型

7. A single fiber of internal radius R and length L is depicted in the following figure. The fiber wall is a membrane which permits diffusional transport of solutes between the liquid solution flowing inside the fiber and an external solution, the dialysate. By adjusting the composition of the dialysate, solutes can be added to or removed from the internal solution as desired. By neglecting the resistance of the dialysate, derive the bulk concentration of solute in the internal solution (C_{ib}) as a function of k_{ci} and k_{mi}, where k_{ci} and k_{mi} are the mass transfer coefficient of solute i through the membrane and the permeability of the membrane to i (analogous to a mass transfer coefficient), respectively. (20%)

Hint: $C_{ib}(z) = \dfrac{\int_A C_i v_z dA}{U}$,

where U is the mean velocity



8. A solute is recovered from a dilute aqueous solution by counter current contact with a pure solvent to remove 98% of the solute. The distribution coefficient of the solute in the system is K_D= **8,** where K_D is defined as the concentration ratio of the solute in the solvent (y) and raffinate phase (x), $K_D = y/x$.
   (a) For this separation requirement, what is the minimum flow rate of solvent (V_{min}) per 100 mole of liquid feed (L), and what is the number of ideal stages required at this condition ? (5 %)
   (b) If the flow rate ratio of solvent (V) to aqueous solution (L) is V:L = 1:2, determine the required number of ideal stages. (8 %)

9. (a) Please write down the definitions of overall efficiency (η_o), Murphy efficiency(η_M), and local efficiency(η') for a distillation column (please describe with the help of a drawing)。(3%)
   (b) Assume constant molar flow rate, please derive the following relationship between η_o and η_M, where L and V are molar flow rate of vapor and liquid phase, respectively. The equilibrium relationship between concentration of vapor (y) and liquid (x) phases is: $y = mx$. (9%)

$$\eta_0 = \frac{\ln\left[1+\eta_M\left(\frac{mV}{L}-1\right)\right]}{\ln\left(\frac{mV}{L}\right)}$$

Kremser Equation:
$$N = \frac{\ln\left[(x_a - x_a^*)/(x_b - x_b^*)\right]}{\ln\left[(x_a - x_b)/(x_a^* - x_b^*)\right]} = \frac{\ln\left[(y_b - y_b^*)/(y_a - y_a^*)\right]}{\ln\left[(y_b - y_a)/(y_b^* - y_a^*)\right]}$$