

※ 考生請注意：本試題可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

Equations

Equation of continuity in cylindrical coordinates

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (\rho r v_r) + \frac{1}{r} \frac{\partial}{\partial \theta} (\rho v_\theta) + \frac{\partial}{\partial z} (\rho v_z) = 0$$

Navier-Stokes equation in cylindrical coordinates

r direction

$$\begin{aligned} \rho \left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} \right) \\ = - \frac{\partial P}{\partial r} + \rho g_r + \mu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r v_r) \right) + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2} \right] \end{aligned}$$

\theta direction

$$\begin{aligned} \rho \left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z} \right) = - \frac{1}{r} \frac{\partial P}{\partial \theta} + \rho g_\theta \\ + \mu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r v_\theta) \right) + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2} \right] \end{aligned}$$

z direction

$$\begin{aligned} \rho \left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right) \\ = - \frac{\partial P}{\partial z} + \rho g_z + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right] \end{aligned}$$

Shear stress components in cylindrical coordinates

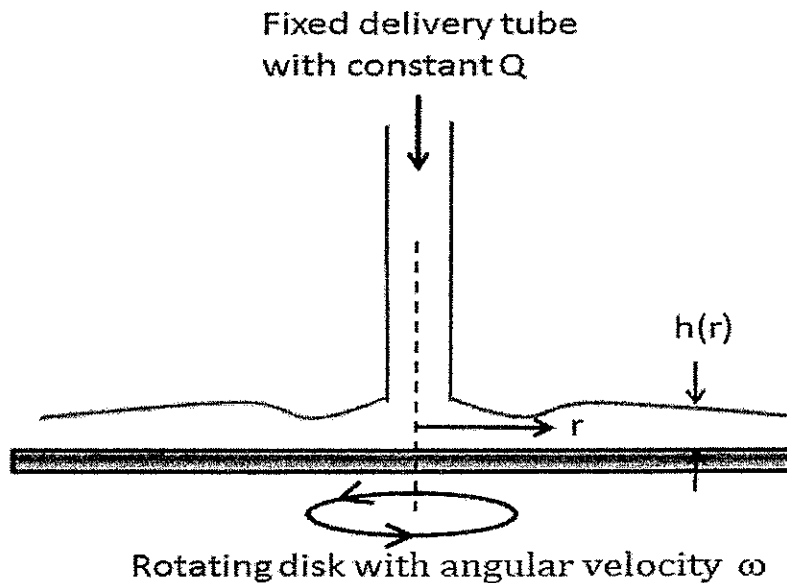
$$\begin{aligned} \tau_{r\theta} = \tau_{\theta r} &= \mu \left[r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right] \\ \tau_{z\theta} = \tau_{\theta z} &= \mu \left[\frac{\partial v_\theta}{\partial z} + \frac{1}{r} \frac{\partial v_z}{\partial \theta} \right] \\ \tau_{zr} = \tau_{rz} &= \mu \left[\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right] \end{aligned}$$

Problems

- Which of the following description(s) is(are) correct? (16%)
 - The physical law underlying the Navier-Stokes equation is the Newton's second law of motion.
 - If a fluid is steady and compressible, the Continuity equation becomes $\nabla \cdot \bar{v} = 0$.
 - If the Reynolds number is large ($\gg 1$), it means the viscous force is dominating in the system.
 - Euler's equation, $\rho \frac{D\bar{v}}{Dt} = \rho \bar{g} - \nabla p$, is applicable for both inviscid fluid flow and irrotational flow.
 - For an incompressible laminar flow, the friction factor depends upon the Reynolds number and pipe

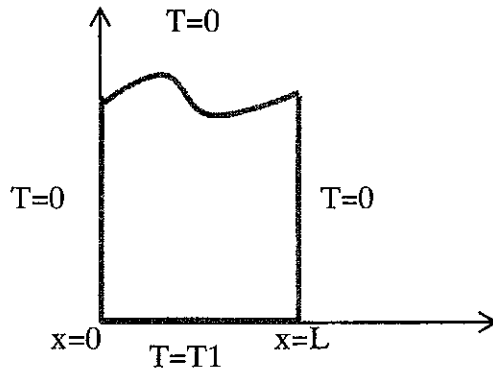
roughness.

- (f) If the flow follows $\nabla \times \vec{v} = 0$, it means there is no viscous force acting on the fluid.
- (g) For a pump used to transport a fluid, its required horse power can be calculated based on conservation of momentum.
- (h) The equivalent length, L_{eq} , is the length of pipe that produces a head loss equivalent to the head loss in a particular fitting.
2. Consider a spin coating process, the coating fluid with a constant volumetric flowrate Q is fed to the axis of a rotating disk from a small tube centered at the axis of rotation as shown below. A thin film was deposited on the rotating disk through this process. The coating fluid with density ρ and viscosity μ is largely flowing in the radial direction. Assume the flow is laminar, steady, and axisymmetric with respect to θ direction. No-slip condition is applied on the surface of the rotating disk. Ignore the gravitational and entrance effects.
- (a) Derive the fluid velocity distribution. (8%)
- (b) Derive the film thickness. (6%)



3. Explain the following: (15%)
- (a) Newton's law of cooling (or the Newton rate equation)
- (b) Stefan-Boltzmann law in thermal radiation
- (c) When analyzing an unsteady-state heat conduction situation, what needs to be checked before applying the lumped-parameter analysis?
- (d) Number of heat transfer unit (in heat exchanger)
- (e) For pool boiling in water on a horizontal wire, why does the heat flux decrease when boiling passes right after the nucleate boiling regime?

4. Consider a two dimensional conduction in an infinitely long rectangular plate as shown in the following. Assume the temperature is a function of x and y only and can be expressed as $T(x, y) = X(x)Y(y)$,
- (a) Write down the governing equation and the boundary conditions. (4%)
- (b) Derive the temperature distribution in the plate. (6%)



5. Briefly explain the following:
- (a) What is the definition of the Schmidt number? (4%)
- (b) Consider the mass transfer of solute A from a solid to a fluid flowing past the surface of the solid. Starting from the mass transfer between the surface and the fluid, show that the Sherwood number equals to the mass-transfer Nusselt number. (8%)
- (c) In the exact solution for the concentration boundary layer developed by Blasius, what is the assumption of the Blasius' approach? (4%)
- (d) The assumption of the Blasius solution fails to predict all kinds of the concentration boundary layer, and Pohlhausen modified the Blasius' method to describe the concentration boundary layer. What is the assumption made by Pohlhausen? (4%)
6. (a) A liquid-liquid extraction process was performed by a spray tower extraction column. To improve the extraction performance, one proposed a method by increasing the length of the column. Is this method effective? Please explain why? (3%)
- (b) Please propose two other methods which can effectively enhance the extraction performance of the tower. (5%)

7. A mixture of benzene and toluene is separated by a distillation column at atmosphere. In the stripping section as shown in the attached figure, the **bottom product** has a flow rate (B) of 50 mole/min and molar fraction of benzene (X_B) is 0.01. It is known that the heat required for the re-boiler is $q_r = 795500$ cal/min.
- Calculate the liquid (L_b) and vapor (V_b) flow rate at the bottom plate of the distillation column. (4%)
 - For considering the variation of flow rate in the stripping section, please write down the required equations. (3%)
 - For a plate in the mediate region of the column, the liquid concentration is $X_m=0.3$. By considering the variation of the flow rate, please estimate the concentration of vapor phase in this plate (y_{m+1}) and the flow rates of liquid (L_m) and vapor (V_{m+1}). (10%)

Please use the data given in the attached figure and table for your calculation. For calculating the enthalpy, the **reference temperature is set at 80 °C**. The ΔH_v of toluene at 80 °C is 8174 cal/gmole.

	ΔH_v at T_b (cal/gmole)	C_{pl} , cal/mole °C	C_{pv} , cal/mole °C	Boiling point (°C)
Benzene	7360	33	23	80
Toluene	7960	40	33	110.6

