

※ 考生請注意：本試題可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

Equations

Equation of continuity in cylindrical coordinates

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (\rho r v_r) + \frac{1}{r} \frac{\partial}{\partial \theta} (\rho v_\theta) + \frac{\partial}{\partial z} (\rho v_z) = 0$$

Navier-Stokes equation in cylindrical coordinates

r direction

$$\rho \left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} \right) = - \frac{\partial P}{\partial r} + \rho g_r + \mu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r v_r) \right) + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2} \right]$$

\theta direction

$$\rho \left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z} \right) = - \frac{1}{r} \frac{\partial P}{\partial \theta} + \rho g_\theta + \mu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r v_\theta) \right) + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2} \right]$$

z direction

$$\rho \left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right) = - \frac{\partial P}{\partial z} + \rho g_z + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right]$$

Shear stress components in cylindrical coordinates

$$\tau_{r\theta} = \tau_{\theta r} = \mu \left[r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right]$$

$$\tau_{z\theta} = \tau_{\theta z} = \mu \left[\frac{\partial v_\theta}{\partial z} + \frac{1}{r} \frac{\partial v_z}{\partial \theta} \right]$$

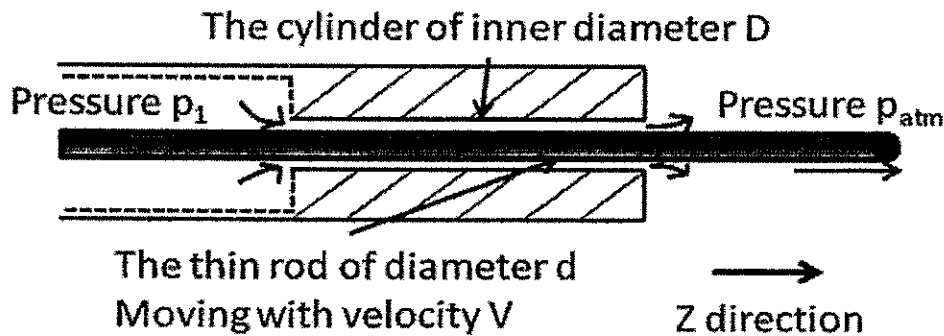
$$\tau_{rz} = \tau_{zr} = \mu \left[\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right]$$

Problems

- There is a fluid in a cylindrical container. Once a solid sphere was put in the container, it would fall down in the fluid. The density of the solid sphere is higher than that of the fluid.
 - It was found that the solid sphere would reach a terminal velocity. Why? (6%)
 - Based on this setup, describe how to use a reference fluid with known viscosity, a solid sphere and a timer to determine the viscosity of the fluid. (6%)

2. Consider a wire-coating process as shown below, in which the cylindrical rod is being moved with a velocity V . The rod is at the center of the cylindrical die. The fluid filling the space between the rod and the inner cylinder wall has density ρ and viscosity μ . Assume the flow is laminar and steady. No-slip condition is applied on the surface of the cylinder and rod. The pressure in the die is p_1 , which is higher than the outside pressure, p_{atm} . Ignore the effect of gravity.

- Derive the differential momentum balance in the z direction. (7%)
- Derive the fluid velocity distribution in the space between the rod and the inner cylinder wall. (7%)
- Derive the shear force acting on the solid cylinder. (4%)



- What is the lumped system analysis in heat transfer analysis? (3%) What is the criterion for applying the lumped system analysis? (2%) A thermocouple is used to measure the temperature of a gas stream. If the junction of the thermocouple is approximated as a sphere with 6 mm in diameter, can the lumped system analysis be applied? (5%) Conductivity, $k=2 \text{ W/m}\cdot\text{°C}$, density, $\rho=8500 \text{ kg/cm}^3$, specific heat, $C_p=320 \text{ J/kg}\cdot\text{°C}$, convection heat transfer coefficient, $h=300 \text{ W/m}^2\cdot\text{°C}$
- 300 °C steam flows through a pipe with inner and outer diameters being 5 and 6 cm, respectively. The conductivity of the pipe is 100 W/m·°C. The pipe is covered with a 3-cm thick insulation with conductivity equal to 0.1 W/m·°C. The heat transfer coefficients outside the insulation and inside the pipe are 20 and 50 W/m²·°C, respectively. The temperature of the surroundings is 15 °C. Determine:
 - Rate of heat loss from the steam per unit length of pipe (10%)
 - Temperature drops across the pipe shell and insulation (5%)
- A graphite rod with a length of 30 cm and a diameter of 2 cm is inserted into a flowing air stream at 1000 K and 3 atm total pressure. The flowing gas creates a stagnant gas boundary layer 5 mm thick around the external surface of the rod. At this temperature, the solid carbon oxidizes to carbon dioxide. Outside of the gas film, the bulk composition of the air stream prevails.
 - Starting from the mole balance of oxygen, derive the differential equation for the concentration (mole fraction) profile of oxygen at various distances away from the center of the rod. (6%)
(Hint: You may ignore the flux of oxygen through θ - and z -direction.)

- (b) The diffusivity of oxygen in the gas mixture is assumed to be $1.0 \times 10^{-5} \text{ m}^2/\text{s}$ and the surface reaction rate constant of oxygen on rod surface is $1.5 \times 10^{-2} \text{ s}^{-1}$. Find the rate of carbon dioxide production from the rod, assuming that the surface reaction is diffusion limited. (7%)
- (c) Estimate the rate of oxygen consumption from the rod, assuming that the surface reaction is reaction limited. (7%)

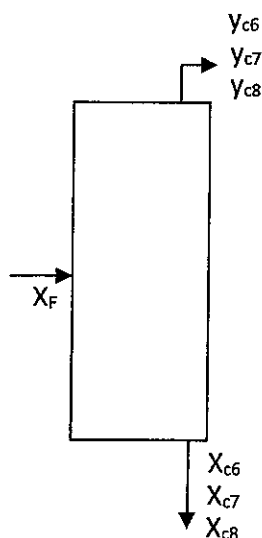
6. A **flash distillation** column was used to separate a mixture containing Hexane (C_6), Heptane (C_7), and Octane (C_8) at **1.2 atm**. For 1 mole of feed, f mole is vaporized as vapor. The feed composition is: $X_{C_6} = 0.4$, $X_{C_7} = 0.27$, $X_{C_8} = 0.33$. The equilibrium parameter, K factor, is defined as $K_i = y_i/x_i$.

- (a) Please derive the equation, shown on the right side, describing the concentration relationship of feed

(X_F) and bottom liquid product (X_B) (6%)

$$x_{Bi} = \frac{x_{Fi}}{f(K_i - 1) + 1}$$

- (b) If $f=0.7$, a student calculated the column temperature and an answer of $105 \text{ }^\circ\text{C}$ was determined. **Please check the correction of this column tempertaure** and calculate the compositions of liquid and vapor products. At $105 \text{ }^\circ\text{C}$, the K_i values of the three components are shown in the attached table. (6%)
- (c) To what temperature must the feed be heated? (The ΔH_v is independent of the temperature) (5%)



	X_{Fi}	K_i at $105 \text{ }^\circ\text{C}$	C_{pl} , cal/mole $^\circ\text{C}$	ΔH_v , cal/mole
C_6	0.4	2.23	62	6370
C_7	0.27	1.04	70	7510
C_8	0.33	0.46	78	8560

7. A continuous fractionation column is designed to separate a liquid mixture with a concentration of X_F and constant thermal condition. The concentrations (X_D , X_B) and producing rates (D , B) of the products are fixed. In this operation, the reflux ratio (R_D) is a variable parameter. Please answer the following questions:
- (a) How to determine the **minimum reflux ratio** (R_{Dm}) for this system. (3%) (Please describe your answer with the help of a figure)
- (b) How to determine the **optimal reflux ratio** (5%) (Please describe your answer with the help of a figure if necessary).