國立成功大學75學年度化學工程研究所考試(工 程 數 學 試題) 共 一 頁 第 一 頁

1. Evaluate $\int_0^a r\{J_n(\alpha r)\}^2 dr$

where $J_n(\alpha r)$ is the nth-order Bessel function of first kind and $[dJ_n(\alpha r)/dr]_{r=a} = 0$. (10%)

2. A tank contains 15 ft^3 of water. A stream of brine containing 2 lb/ft^3 of salt is fed into the tank at a rate of 3 ft^3/min.. Liquid flows from the tank at a rate of 4 ft^3/min.. If the tank is well agitated, what are the salt concentrations in the tank when the tank contains: (a) 5 ft^3 of brine, and (b) the last drop of brine? (15%)

3. Define $\operatorname{erfc} \eta = \frac{2}{\sqrt{\pi}} \int_\eta^\infty e^{-x^2} dx$, evaluate $\int_0^\infty \operatorname{erfc} \eta \, d\eta$ (10%)

4. Solve $4x^2 \frac{d^2y}{dx^2} + y = x^{1/2}$ (10%)

5. How to find the appropriate value of β in $\eta = (x/\sqrt{4D})t^\beta$ in order to reduce the partial differential equation: $\partial c/\partial t = D\,\partial^2 c/\partial x^2$ into an ordinary differential equation of $C(\eta)$. D is a constant. This is the method of so-called combination of variables. Solve $c(x,t)$ completely by this method with the use of conditions: at $t=0$, $c=0$; at $x=0$, $c=c_0$; at $x=\infty$, $c=0$. (15%)

6. Evaluate $\int_0^\infty \frac{\cos x}{1+x^2} dx$ by the method of Laplace transform. (10%)

7. (a) Expand x^2, in the interval $-\pi$ to π, into the form

$$x^2 = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos nx + \sum_{n=1}^{\infty} B_n \sin nx$$

(b) Calculate the following sequence by the use of (a): $1/1^2 + 1/3^2 + 1/5^2 + \cdots$ (15%)

8. The acceleration vector of a moving particle is

$$\bar{a} = 12\cos 2t\,\bar{i} - 8\sin 2t\,\bar{j} + 16t\,\bar{k}$$

Initially ($t=0$), the particle is at the origion (0,0,0) and is stationary ($\bar{V}=0$). Find its position and velocity vectors when $t=\pi$. (15%)