516 (1)

(甲組)

國立成功大學76學年度化工程研究所考試(工程數學 試題) 共2頁 第1頁

(甲組)

1. Find a general solution of the equations:

(a) $(D^2+4)y = \cos 2x + \cos 4x$ (5%)

(b) $4x^2y''+4xy'+(x-\nu^2)y=0$, where ν is real and nonnegative (5%)

2. (a) For a function $f(z)=u(x,y)+i\,v(x,y)$ to be analytic, the Cauchy-Riemann equations should be satisfied, what are the equations? (5%)

(b) Show that if the equation $u(x,y)=c=$ constant represents a family of curves, its orthogonal trajectories can be represented by the equation $v(x,y)=c^*=$ constant (5%)

(c) Determine the orthogonal trajectories of $x^2-y^2=c$

3. Evaluate $\displaystyle\int_{-\infty}^{\infty}\frac{\sin x}{x}\,dx$ (5%)

(10%)

4. Find the steady-state solution, i.e., the solution as t approaches infinity, of the differential equation

$$y''+2y'+10y=r(t)$$

where

$$r(t)=\begin{cases}1 & (0<t<\pi)\\ -1 & (\pi<t<2\pi)\end{cases},\quad r(t+2\pi)=r(t)$$

(15%)

5. Solve the system of differential equations

(a) $\dot{x}=-2x+y$

(b) $\dot{y}=-4x+3y+10\cos t$

by using vectors and matrices

(15%)

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6. (a) Evaluate directly the line integral $\int_C v_t\, ds$, where $\vec{v} = (-y\vec{i} + x\vec{j})/(x^2+y^2)$, $C: x^2+y^2=1$ ($z=0$), v_t is the component of $\vec{v}$ in the direction of the unit tangent vector of C, oriented in the counterclockwise sense. (5%)

(b) Does this line integral be independent of path? Why? (5%)

(c) Can the Stokes's theorem be applied? Why? (5%)

7. (a) Find the temperature $u(x,t)$ in a bar of length l which is perfectly insulated, also at the ends at $x=0$ and $x=l$, assuming that $u(x,0)=f(x)$. (15%)

(b) In (a), what will the temperature be, as $t \to \infty$? Does this agree with your physical intuition? (5%)