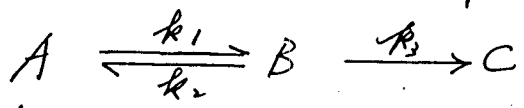


2/6 (2.解)

1. Most radioactive substances disintegrate at a rate proportional to the amount of substance instantaneously present. The time in which 50% of a given amount disappears is called the half-life of the substance. For ${}^{226}\text{Ra}$ the half-life is $H=1600$ years. What percentage will disappear in 10 years? (10%)

2. It is desired to produce a substance B from a raw material A in a continuous stirred tank reactor of effective volume $V \text{ m}^3$. If $Q \text{ m}^3/\text{s}$ of a solution of A of concentration C_0 is fed to the empty reactor, and the chemical reaction of A is represented by



in which all the reactions are of first order, set up a differential equation to describe the number of moles of B as function of time in the initial discharge from the reactor (15%)

3. Solve the system of differential equations:

$$\begin{aligned} \dot{y}_1 &= 5y_1 + 8y_2 + 1, & y_1(0) &= 4 \\ \dot{y}_2 &= -6y_1 - 9y_2 + t, & y_2(0) &= -3, \end{aligned} \text{ where } \dot{y} = dy/dt \text{ (15\%)}$$

4. Solve the partial differential equation:

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad c^2 = \text{positive constant}$$

$$\frac{\partial u}{\partial x}(0, t) = 0, \quad u(l, t) = 0, \quad \text{for all } t$$

$$u(x, 0) = f(x)$$

(20%)

(乙組)

5. In an experiment the experimenter selects n values x_1, x_2, \dots, x_n of x and then observes y at those values of x , so that he obtains a sample of the form (x_i, y_i) , $i=1, 2, \dots, n$. If a linear relationship between y and x is assumed, how can he obtain the "best" fitted equation? (15%)

6. Show that for the case where the function $f(x, y, z)$ to be made a maximum or minimum subject to the constraint condition $\phi(x, y, z) = 0$, the method of Lagrange multipliers can be used:

Form the function $h(x, y, z) = f(x, y, z) + \lambda \phi(x, y, z)$ and set

$$\frac{\partial h}{\partial x} = 0, \quad \frac{\partial h}{\partial y} = 0, \quad \frac{\partial h}{\partial z} = 0$$

where the constant λ is called a Lagrange multiplier (15%)

7. Express in cylindrical and spherical coordinates
(a) grad f (b) $\text{div } \vec{v}$ (c) $\text{curl } \vec{v}$ (d) $\nabla^2 f$

(10%)