(乙組)

國立成功大學 76 學年度化學工程研究所碩士班入學考試(應用數學試題) 共 2 頁 第 1 頁

1. Most radioactive substances disintegrate at a rate proportional to the amount of substance instantaneously present. The time in which 50% of a given amount disappears is called the half-life of the substance. For $_{88}Ra^{226}$ the half-life is H = 1600 years. What percentage will disappear in 10 years? (10%)

2. It is desired to produce a substance B from a raw material A in a continuous stirred tank reactor of effective volume V m^3. If Q m^3/s of a solution of A of concentration C_0 is fed to the empty reactor, and the chemical reaction of A is represented by

$$A \underset{k_2}{\overset{k_1}{\rightleftharpoons}} B \xrightarrow{k_3} C$$

in which all the reactions are of first order, set up a differential equation to describe the number of moles of B as function of time in the initial discharge from the reactor (15%)

3. Solve the system of differential equations:

$$\dot{y}_1 = 5y_1 + 8y_2 + 1, \quad y_1(0) = 4$$
$$\dot{y}_2 = -6y_1 - 9y_2 + t, \quad y_2(0) = -3, \quad \text{where } \dot{y} = dy/dt$$

(15%)

4. Solve the partial differential equation:

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad c^2 = \text{positive constant}$$
$$\frac{\partial u}{\partial x}(0,t) = 0, \quad u(l,t) = 0, \text{ for all } t$$
$$u(x,0) = f(x)$$

(20%)

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5. In an experiment the experimenter selects n values $x_1, x_2, \cdots, x_n$ of x and then observes y at those values of x, so that he obtains a sample of the form (x_i, y_i), $i = 1, 2, \cdots, n$. If a linear relationship between y and x is assumed, how can he obtain the "best" fitted equation? (15%)

6. Show that for the case where the function $f(x, y, z)$ to be made a maximum or minimum subject to the constraint condition $\phi(x, y, z) = 0$, the method of Lagrange multipliers can be used:

   Form the function $h(x, y, z) = f(x, y, z) + \lambda\phi(x, y, z)$ and set

   $$\partial h/\partial x = 0, \quad \partial h/\partial y = 0, \quad \frac{\partial h}{\partial z} = 0$$

   where the constant λ is called a Lagrange multiplier. (15%)

7. Express in cylindrical and spherical coordinates
   (a) grad f (b) div $\vec{v}$ (c) curl $\vec{v}$ (d) $\nabla^2 f$

   (10%)