1. A square matrix $A = [a_{ij}]$ is Hermitian provided $\bar{a}_{ij} = a_{ji}$ for all
   values of i and j, where $\bar{a}_{ij}$ is the conjugate of a_{ij}.

   (a) Prove: If A and B are real n-square matrices with A non-
       singular and if $H = A + iB$ ($i = \sqrt{-1}$) is Hermitian, then
       $$|H|^2 = |A|^2 \ |I + (A^{-1}B)^2| \qquad (6\text{分})$$

   (b) Find the trace, adjoint and inverse of the matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ (6分)

2. (a) Prove that xy^2 cannot be the real part of an analytic function
       of z. (5分)

   (b) Expand the function $f(z) = 1/[z(1-z)]$ in a Laurent series which
       converges in the region $0 < |z| < 1$. (6分)

   (c) If $f(z) = \exp(z)$, find a complex variable c such that $c \neq 0$ and
       $f(z+c) = f(z)$. (5分)

3. (a) Determine the value of the surface integral $\iint_S \vec{F} \cdot \vec{n} \, d\sigma$, where
       $\vec{F} = xy\vec{i} + xz\vec{j} + (1-z-yz)\vec{k}$ and S is the portion of the paraboloid
       $z = 1 - x^2 - y^2$ for which $z \geqslant 0$. (6分)

   (b) Find the volume of the tetrahedron with vertices at the points
       $(0, 0, 0)$, $(2, 1, 1)$, and $(1, 2, 1)$. (5分)

   (c) Show that if $\nabla \times \vec{F} = \vec{0}$, then the line integral of $\vec{F}$ once around a
       closed curve C is zero, that is
       $$\oint_C \vec{F} \cdot d\vec{r} = 0. \qquad (5\text{分})$$

4. If $E = f(p,T)$ and $T = g(p,v)$, show that
   $$\left(\frac{\partial E}{\partial p}\right)_v = \left(\frac{\partial E}{\partial p}\right)_T + \left(\frac{\partial E}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_v. \qquad (6\text{分})$$

5. Show how can you reduce the nonlinear differential equation
   $$P(x)y' + Q(x)y = R(x)y^a \quad \text{where a is any real number,}$$
   to a linear one. (5分)

6. (a) Solve the problem
       $y'' - 3y' + 2y = 4x + 3e^{3x}$, $y(0)=3$, $y'(0)=3$
       by (1) one of the standard methods; (2) the Laplace transform method. (10分)

   (b) What are the advantages of the Laplace transform method over other
       standard method for obtaining the particular solution of the nonhomogeneous
       differential equation. (5分)

7. (a) Let $g_1(x)$, $g_2(x)$,..... be any orthogonal set of functions on an interval $a \le x \le b$ and let $f(x)$ be a given function which can be represented in terms of the g_j's by a convergent series

$$f(x) = \sum_{n=1}^{\infty} c_n g_n(x).$$

Show how can you determine the constants c_n. (5分)

(b) Which orthogonal set of functions yields Fourier series? Derive the Euler formulas. (5分)

8. Solve the following partial differential equation by the method of separating variables

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$u(0,t) = 0$

$u_x(\ell, t) = 0$, $u_x = \partial u / \partial x$

$u(x,0) = u_0 = $ constant, $0 \le x \le \ell$

(a) Find the eigenvalues and eigenfunctions of the corresponding Sturm-Liouville problem. (10分)

(b) Obtain an expression for $u(x,t)$. (10分)