

1. A reactor having only five rectangular sides is required to have a given volume V . Determine its dimensions so that the surface area will be a minimum. (10%)

2. The absorption of light in a very thin transparent layer is proportional to the thickness of the layer and to the amount incident on that layer. Formulate this in terms of a differential equation and solve it. (10%)

3. The relationship between two variables, i.e. x and y , is

$$y = ax^b.$$

Show how can you determine the coefficients a and b experimentally. (10%)

4. The temperature distribution in a long thin bar of constant cross section and homogeneous material, which is oriented along the x -axis and is perfectly insulated laterally, is governed by the one-dimensional heat equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad c^2 = K/\sigma\rho$$

where K is the thermal conductivity, σ is the specific heat, and ρ is the density of the material. If the ends $x=0$ and $x=l$ of the bar are kept at temperature zero, and $f(x)$ is the initial temperature in the bar. Determine the temperature $u(x,t)$. (20%)

5. By means of line integral, find the area of the interior of the cardioid $r = a(1 - \cos\theta)$, $0 \leq \theta \leq 2\pi$. (10%)

6. Find the volume of the tetrahedron that has the following vertices $(0,1,2)$, $(5,5,6)$, $(1,2,1)$, $(3,3,1)$ (10%)

7. Show how can you reduce the nonlinear differential equation $P(x)y' + Q(x)y = R(x)y^a$ where a is any real number, to a linear one. (5%)

8. (a) Solve the problem

$$y'' - 3y' + 2y = 4x + 3e^{3x}, \quad y(0)=3, \quad y'(0)=3 \text{ by}$$

(1) one of the standard methods; (5%)

(2) the Laplace transform method. (5%)

(b) What are the advantages of the Laplace transform method over other standard method for obtaining the particular solution of a nonhomogeneous differential equation? (5%)

9. (a) Let $g_1(x), g_2(x), \dots$ be any orthogonal set of functions on an interval $a \leq x \leq b$ and let $f(x)$ be a given function which can be represented in terms of the g_j 's by a convergent series

$$f(x) = \sum_{n=1}^{\infty} c_n g_n(x).$$

Show how can you determine the constants c_n . (5%)

(b) Which orthogonal set of functions yields Fourier series? Derive the Euler formulas. (5%)