1. It can be shown that, if the temperature is kept constant, the velocity of a chemical reaction is proportional to the product of the concentrations of the substances which are reacting. A bimolecular reaction A + B → M combines a moles per liter of a substance A and b moles of a substance B. If y is the number of moles per liter that have reacted after time t, derive the rate equation and solve it, assuming that $a \neq b$. (15%)

2. Find (a) Taylor series representation of sinx. (b) Fourier series representation of $f(x)=2kx/L$ if $0<x<L/2$; $2k(L-x)/L$ if $L/2<x<L$. (c) Fourier integral representation of $f(x)= \exp(-kx)$, $x>0$, $k>0$. (15%)

3. One way of simulating a process is to subject the process to a given input, and to measure the respective output. Typical inputs used in practice are unit step function, DiracDelta function, and sinusoidal functions. What are the Laplace transforms of these inputs, respectively? (15%)

4. Consider the cardioid $r=a(1-\cos\theta)$, where $0 \leq \theta \leq 2\pi$. Find the arc length and the area of of the interior of the cardioid. (10%)

5. Find the eigenvalues and eigenfunctions of the Sturm-Liouville problem:
$y'' + \lambda y = 0$. $y'(0)=0$, $y'(\pi)=0$. (15%)

6. Find a general solution of equation(a) and equation(b) in terms of Bessel functions and Legendre polynomials, respectively:
(a) $y'' + (1/x)y' + k^2 y = 0$. (b) $(1/\sin x)d(\sin x\, dy/dx)/dx + n(n+1)y = 0$. $n=0,1,2,..$ (20%)

7. Determine the dimensions of the largest rectangular parallelepiped which can be inscribed in a hemisphere of radius R. (10%)