1 (6%) Solve for the general solution

$$x \sin 2y\, dx + (x^2 \cos 2y + 3y)\, dy = 0$$

2 (10%) Solve the following system of differential equations and determine the type and stability behavior of the critical point.

$$\begin{cases} x' = x + y \\ y' = 4x + y \end{cases} \qquad \text{where} \quad \text{“}' \text{”} = \frac{d}{dt}$$

3 (10%) Find the Laplace transform of the function

$$f(t) = \begin{cases} 1 & \text{if} \quad 0 < t < 1 \\ 0 & \text{if} \quad 1 < t < 2\pi \\ \cos t & \text{if} \quad t > 2\pi \end{cases}$$

4 (8%) Find the eigenvalues and eigenvectors of the following matrix.

$$\begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5 (12%) Evaluate the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n}\, dA$ by the divergence theorem, where

$$\mathbf{F} = x y^2 \mathbf{i} + y^3 \mathbf{j} + 4 x^2 z \mathbf{k}$$

and S is the surface of the cylinder $x^2 + y^2 \le 4$, $0 \le z \le 5$.

6 (10%) Solve $x^2 y'' + 2 x y' - 2 y = 6x$ with $y(1) = 3$, $y'(1) = -7$.

7 (12%) Find the solution to the following differential equation in terms of Bessel's functions.

$$y'' + e^{2x} y = 0 \qquad (\text{Hint: let } e^x = u)$$

8 (12%) Using the indicated transformations, solve the following partial differential equation where u = u(x, y)

$$x\,u_{xy} = y\,u_{yy} + u_y \qquad (v = x,\ z = x\,y)$$

9 (10%) Please write down the Lagrange's interpolation formula for the fitted curve passing through the following four points:

$$(x_0, y_0), (x_1, y_1), (x_2, y_2), (x_3, y_3)$$

In addition, what are the deficiencies of the Lagrange's interpolation method.

10 (10%) Find the tangential acceleration and the normal acceleration of the motion given by $\mathbf{r}(t) = t\,\mathbf{i} - t^2\,\mathbf{j}$