1 (12%) Find the Fourier series of f(x), which has the period 8:

$$f(x) = \begin{cases} 0, & \text{if } -4 < x < -2 \\ 1, & \text{if } -2 < x < 2 \\ 0, & \text{if } 2 < x < 4 \end{cases}$$

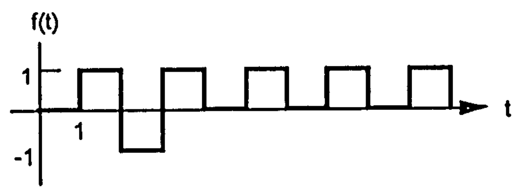
2 (8%) Find the radius of convergence and convergence interval of the series

$$\sum_{m=0}^{\infty} \frac{(x-2)^{2m}}{9^m}$$

3 (12%) Solve the following system of differential equations:

$$\begin{cases} x' = 2x + 3y \\ y' = \dfrac{x}{3} + 2y \end{cases}$$

4 (12%) Represent the following function, f(t), in terms of unit step functions and find the Laplace transform of f(t).



5 (8%) Solve $y(t) = 2 + 2\int_0^t y(\tau)\,d\tau$

6 (a) (6%) If f(x, y, z) = 0 determines z = z(x, y), show that

$$\frac{\partial z}{\partial x} = -\frac{\dfrac{\partial f}{\partial x}}{\dfrac{\partial f}{\partial z}} \qquad \text{if } \frac{\partial f}{\partial z} \neq 0$$

(b) (6%) For the van der Waals equation of a gas

$$\left(p + \frac{a}{V^2}\right)(V - b) = R'T$$

where p is the pressure, T is the absolute temperature, V is the specific volume of the gas, and a, b, R' are constants, determine $\left(\dfrac{\partial V}{\partial p}\right)_T$.

7 (8%) A B 兩城市的居民相互遷移，每一年城市 A 中有 20% 之人口遷往城市 B，城市 B 中有 10% 之人口遷往城市 A；假設 A 與 B 的總人口數不變(無出生死亡等)，原先 A 市有 150 萬人，B 市有 60 萬人，試建立一 Matrix model 描述經過 n 年後 A 市與 B 市之人口。並請計算在上述 matrix model 中之 matrix，其 eigenvalues 及 eigenvectors。

8 (8%) Find the tangent plane and the normal line to the following surface S at the given point P:

$$S:\ z = x^2 - 2y + 4, \qquad P = (1, 1, 3)$$

9 (10%) 在直角座標中，三個 vectors a, b, 與 c，我們可得

$$\mathbf{b}\times\mathbf{c} = \begin{vmatrix} i & j & k \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} \quad ; \quad \mathbf{a}\bullet(\mathbf{b}\times\mathbf{c}) = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

其中 $a_x, a_y, \ldots, c_z$ 等為 a, b, c 在 x, y, z 上之分量。同理，對於任意之 vector function v 而言 (where v is differentiable)，

$$\nabla\times\mathbf{v} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix} ; \quad \mathbf{v}\bullet(\nabla\times\mathbf{v}) = \begin{vmatrix} v_x & v_y & v_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix}$$

但因 $\mathbf{v}\bullet(\nabla\times\mathbf{v})$ 的 determinant 中有兩列相同，由 determinant 定理，我們得

$$\mathbf{v}\bullet(\nabla\times\mathbf{v}) = 0$$

故我們得以下之推論："v and ∇× v are orthogonal."

請問你對以上步驟與推論有何看法？請說明理由。

10 (10%) Find a general solution to

$$(D^3 - 3D^2 + 3D - 1)\,y = e^x\sqrt{x}$$

where $D = \frac{d}{dx}$.