

1. (a) Find the half-life of a radioactive substance from two measurements $y_1=y(t_1)$ and $y_2=y(t_2)$ of the amounts present at times t_1 and t_2 . (10%)
(b) The half-life of ${}^6\text{C}^{14}$ is 5730 years. If a fossilized bone contains 25% of the original amount of radioactive ${}^6\text{C}^{14}$, what is its age? (10%)
2. A tank contains 1 m^3 of water. A stream of brine containing 10 kg/m^3 of salt is fed into the tank at a rate of $0.02\text{ m}^3/\text{s}$. Liquid flows from the tank at a rate of $0.01\text{ m}^3/\text{s}$. If the tank is well agitated, what is the salt concentration in the tank when the tank contains 2 m^3 of brine? (20%)
3. What are the Laplace transforms of the typical input functions: unit step function, Dirac delta function, and sinusoidal function? (10%)
4. Find the volume of the tetrahedron that has the following vertices: $(1,1,1)$, $(2,4,1)$, $(5,1,1)$, $(4,9,3)$. (10%)
5. Consider the cardioid in polar coordinates: $r=a(1-\cos\theta)$, where $0\leq\theta\leq 2\pi$. Find the arc length and the area of the interior of the cardioid. (10%)
6. In a temperature field, heat flows in the direction of maximum decrease of temperature T . Find the direction at $P:(1/4, -2, 1/2)$ when $T=4x^2+y^2-5z^2$. (10%)
7. Find the value of the series $1-1/3+1/5-1/7+\dots$ (10%)
8. A reactor having only five rectangular sides is required to have a given volume V . Determine its dimensions so that the surface area will be a minimum. (10%)