系所組別： 水利及海洋工程學系甲、乙組

考試科目： 工程數學

※ 考生請注意：本試題 □可 ☑不可 使用計算機

1. Find the solution of the initial-value problem (20%)

$$\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 25y = 0$$

with initial conditions: $y(0) = -3$, and $y'(0) = -1$.

2. Solve the fourth-order ordinary differential equation (20%)

$$\frac{d^4y}{dx^4} = 4 \quad \text{for} \quad 0 < x < 1$$

with boundary conditions: $y(0) = 0$, $y''(0) = 0$, $y(1) = 0$, and $y''(1) = 0$

3. Find a general solution of the differential equation (20%)

$$\frac{d^2y}{dt^2} + 36y = r(t)$$

where $r(t)$ is a periodic function, and its definition in a period is given as

$$r(t) = \begin{cases} t + \dfrac{\pi}{2} & \text{if } -\pi < t \le 0 \\ -t + \dfrac{\pi}{2} & \text{if } 0 < t \le \pi \end{cases}$$

4. Find the solution of the vibrating-string problem (20%)

PDE: $\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2}$, with $0 < x < \pi$, and $0 < t < \infty$

ICs: $u_t(x,0) = 0$ and $u(x,0) = \dfrac{3}{4}\sin x - \dfrac{1}{4}\sin 3x$

BCs: $u_t(0,t) = 0$ and $u(\pi,t) = 0$ for all t

5. Solve the linear system (20%)

$$\begin{aligned} -x + y + 2z &= 2 \\ 3x - y + z &= 6 \\ -x + 3y + 4z &= 4 \end{aligned}$$