

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Solve: (a)  $\frac{d}{dx} \ln(e^{3x} + \cos 2x)$ ; (5%)

(20 分) (b)  $\int e^{3x} \cos(e^{3x}) dx$ ; (5%)

(c)  $\int_0^{\infty} x^3 e^{-x} dx$  (hint: Gamma function  $\Gamma(n) = \int_0^{\infty} x^{n-1} e^{-x} dx$ ). (10%)

2. Find: (a) the formal Taylor series for  $e^x$  about  $x = 0$ ; (10%)

(20 分) (b) the approximate value of  $\int_0^1 \frac{1 - e^{-x}}{x} dx$  by using a series method. (10%)

3. Find: (a) the solution of the linear equation  $x \frac{dy}{dx} - 2y = x^3 \cos 4x$ ; (10%)

(20 分) (b) all complex values of  $z$  such that  $e^z = -1$ . (10%)

4. Expand the following function in a Fourier series. (20%)

(20 分)  $f(x) = \begin{cases} 0 & -5 < x < 0 \\ 3 & 0 < x < 5 \end{cases}$  with a period of 10.

5. Solve: (a) the third order differential equation. (10%)

(20 分)  $y''' - 5y'' + 8y' - 4y = 0$  with  $y(0) = 0, y'(0) = 1, y''(0) = -1$ .

(b) the partial differential equation for heat condition in a thin rod of length 2. (10%)

$$u_{xx} - ku_t = 0 \quad \text{with } 0 < x < 2, t > 0 \text{ and } k > 0.$$

If  $u(x, t) = 2 \sin(3\pi x)$  and  $u(0, t) = u(2, t) = 0$ .