

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (20%) (a) Solve the following initial value problem using Laplace transform

$$y'' + y' + y = g(t)$$

$$\text{where } g(t) = \begin{cases} 0, & 0 \leq t < 1 \\ 1, & 1 \leq t \end{cases}, \quad y(0) = 1, \quad y'(0) = 0$$

- (b) What happens to your solution when $t \rightarrow \infty$?

2. (20%) Using the geometric series, find Laurent expansions for

$$f(z) = \frac{1}{(z-1)(z-2)} \quad \text{valid in } |z| < 1 \quad \text{and valid in } |z| > 2.$$

3. (25%) Consider the following problem for vibrations of a circular membrane

$$u_{tt} = c^2(u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta}) \quad 0 < r < a$$

$$u(r = a, \theta, t) = 0$$

$$u(r, \theta, t = 0) = 0.$$

$$u_t(r, \theta, t = 0) = g(r, \theta).$$

4. (20%) (a) If $a \neq c$, find the eigenvalue matrix \mathbf{A} and eigenvector matrix \mathbf{S} in

$$\mathbf{A} = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} = \mathbf{S}\mathbf{\Lambda}\mathbf{S}^{-1}.$$

- (b) Find the four entries in the matrix \mathbf{A}^{105} .

5. (15%) Consider the region R enclosed by the x -axis, $x = 1$ and $y = x^3$

- (a) Use the Green's theorem to find the flux of $\vec{F} = (1 + y^2)\hat{j}$ out of R . (\hat{j} is the unit vector in y direction)

- (b) Find the flux out of R through the two sides C_1 (the horizontal segment) and C_2 (the vertical segment).

- (c) Use parts (a) and (b) to find the flux out of the third side C_3 .