

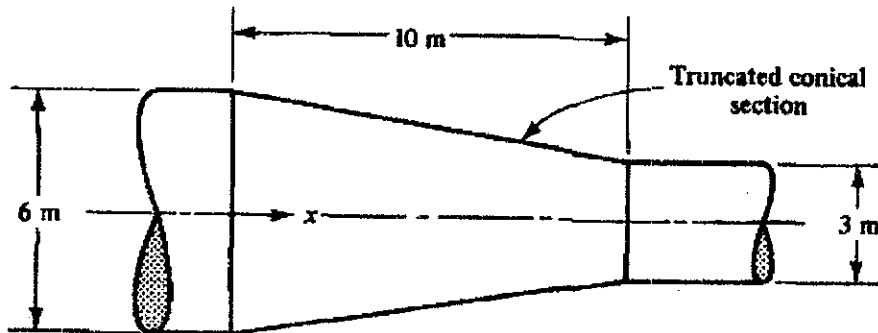
※ 考生請注意：本試題可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (16%) Write a short, qualitative definition of the following terms:
  - (a) Inviscid flow
  - (b) Incompressible flow
  - (c) Irrotational flow
  - (d) Velocity potential
  - (e) Boundary layer
  - (f) Turbulent flow
  - (g) Drag
  - (h) Buckingham Pi Theorem
2. (20%) Calculate the vorticity at any point  $(R, \theta)$  for each of the following two-dimensional flow fields:
  - (a)  $u_R = 0, u_\theta = \Omega R,$
  - (b)  $u_R = 0, u_\theta = \frac{\Gamma}{2\pi R},$
  - (c) Calculate the circulation for case (b) and discuss the results in terms of circulation and vorticity

Where  $R$  and  $\theta$  are cylindrical coordinates,  $\Omega$  (angular velocity) and  $\Gamma$  (circulation) are constants. Note that the in cylindrical coordinates  $(R, \theta, Z)$

$$\nabla \times \mathbf{u} = \left( \frac{1}{R} \frac{\partial(Ru_\theta)}{\partial R} - \frac{1}{R} \frac{\partial u_R}{\partial \theta} \right) \mathbf{e}_z$$

3. (20%) Consider the following transition section:



- (a) The flow from left to right is constant at  $Q = 12\pi \text{ m}^3/\text{s}$ . What is the total acceleration of a water particle in the  $x$  direction at  $x = 5\text{m}$ ? Assume that the water is incompressible and that the  $x$  component of velocity is uniform across each cross section.
- (b) The flow of water from *right to left* is given by  $Q(t) = \pi t^2$   
Calculate the total acceleration at  $x = 5 \text{ m}$  for  $t = 2.0\text{s}$ . Make the same assumptions as in part (a).

4. (20%) If the water (assumed inviscid) in the U-tube is displaced from its equilibrium position, it will oscillate about this position with its natural period. Assume that the displacement of the surface is

$$\eta(t) = A \cos \frac{2\pi}{T}t$$

where the amplitude  $A$  is 10 cm and the natural period  $T$  is 8 s. What will be the pressure at a distance 20 cm below the instantaneous water surface for  $\eta = +10, 0,$  and  $-10$  cm? Assume that  $g = 980 \text{ cm/s}^2$  and  $\rho = 1 \text{ g/cm}^3$ .

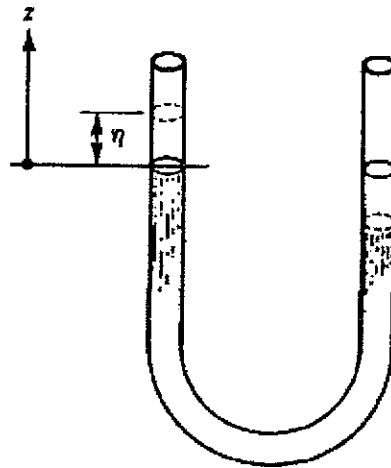


Figure 1: sketch of problem 4

5. (24%) Consider steady flow between two parallel inclined plates, driven by both constant pressure gradient ( $dp/dx$ ) and gravity. The distance between the two plates is  $2H$  and the chosen system of coordinates is shown in Fig. 2. The angle formed by the two plates and the horizontal direction is  $\theta$ . Calculate velocity component in  $x$ -direction (i.e.  $u_x$ ) and pressure.

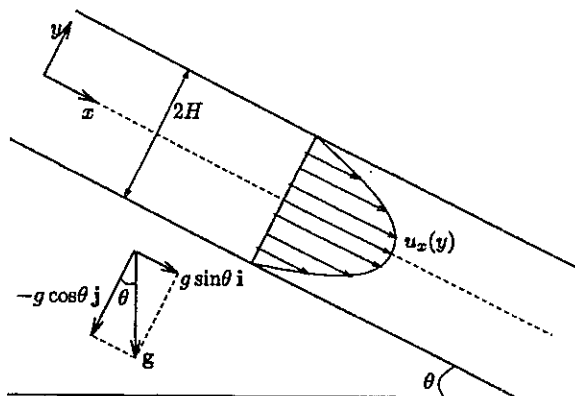


Figure 2: Poiseuille flow between inclined plates