

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (30%) Consider a driven undamped mass-spring system described by the initial-value problem

$$y'' + \omega_0^2 y = K \sin^3 \omega t \text{ where } y(0) = y'(0) = 0$$

(a)(10%) Discuss why there are two frequencies ($\omega = \omega_1, \omega_2$, assuming $\omega_1 > \omega_2$) at which the system is in pure resonance.

(b)(20%) Obtain the solutions of the initial-value problem for $\omega = \omega_1$ and $\omega = \omega_2$.

2. (20%) If $\mathbf{F} = c\nabla(1/r)$, where c is constant and $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ is a position vector with $\|\mathbf{r}\| = r$, find the flux of \mathbf{F} through the sphere $x^2 + y^2 + z^2 = a^2$.

3. (25%) The steady temperatures in the circular ring shown in Figure 1 can be described by the following Laplace equations.

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0.$$

Find the steady-state temperature $u(r, \theta)$.

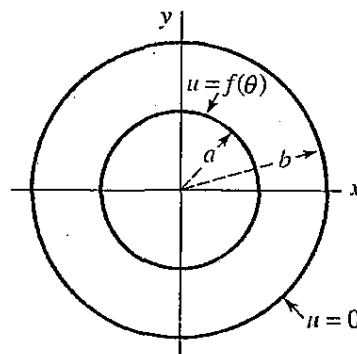


Figure 1: Ring in Problem 3

4. (25%) All of the questions below refer to the following matrix A

$$A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

(a) (5%) What is the rank of A ?

(b) (8%) Find a basis for the nullspace $N(A)$ of A .

(c) (5%) Does there exist a vector $b \in \mathbb{R}^2$ such that $Ax = b$ has no solution?

(d) (7%) Find all solutions of $Ax = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$. Express your solution in the form $x = x_{\text{particular}} + c_1x_1 + c_2x_2$ where x_1, x_2 are special solutions.