

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. [20%] Solve $y'' + \omega^2 y = r(t)$, where $|\omega| \neq 0, 1, 2, \dots$, $r(t)$ should be expressed in Fourier series with 2π period and

[a] $r(t) = 3t^2 \quad (-\pi < t < \pi)$

[b] $r(t) = |t| \quad (-\pi < t < \pi)$

2. [20%] Solve 2D Laplace's equation for a circular disk. Please start from the Laplace equation in the Cartesian coordinate system to that of the polar coordinates.

G.E. $u_{xx} + u_{yy} = 0$, where $\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases}$

B.C. $\begin{cases} u(a, \theta) = f(\theta) \\ u(0, \theta), \text{ bounded} \end{cases}$; periodicity $\begin{cases} u(r, -\pi) = u(r, \pi) \\ \frac{\partial u}{\partial \theta}(r, -\pi) = \frac{\partial u}{\partial \theta}(r, \pi) \end{cases}$

3. [20%] Using the Laplace transform to solve $y'' + 4y = \begin{cases} 8t^2, & \text{if } 0 < t < 5. \\ 0, & \text{if } t > 5. \end{cases}$
 $y(1) = 1 + \cos 2$, $y'(1) = 4 - 2 \sin 2$

4. [20%] Evaluate the integral directly by the divergence theorem (i.e., $\iint_S \vec{F} \cdot \vec{n} \, dA$).

[a] $\vec{F} = [y + z, 20y, 2z^3]$, S is the surface of $0 \leq x \leq 2$, $0 \leq y \leq 1$, $0 \leq z \leq y$

[b] $\vec{F} = [e^x, e^y, e^z]$, S is the surface of the box $|x| \leq 1$, $|y| \leq 1$, $|z| \leq 1$.

5. [20%] Given a matrix $A = \begin{bmatrix} 1-k & 1 & 0 \\ 1 & 1-k & 0 \\ 0 & 0 & 1 \end{bmatrix}$, where k is a constant to be determined.

[a] Determine the relationship between the rank of A and the values of k .

[b] Determine the value of k if the matrix A is an orthogonal matrix.