

國立成功大學
110學年度碩士班招生考試試題

編 號： 105

系 所： 水利及海洋工程學系

科 目： 工程數學

日 期： 0203

節 次： 第 2 節

備 註： 不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Find the solutions of the following differentiations and integrations: (25 分)

(a) $\frac{d}{dx} \ln(\cos 3x + 3e^{3x})$ (5%) ; (b) $\frac{d}{dx} \sqrt{e^{2x} + (\sin x)^2}$ (5%) ;

(c) $\int_0^1 e^{4x} \sin(e^{4x}) dx$ (5%) ; (d) $\int_0^\infty x^5 e^{-x} dx$ (10%)

(Hint: Gamma function $\Gamma(n) = \int_0^\infty x^{n-1} e^{-x} dx$)

2. Solve the following initial value problems: (25 分)

(a) $y'' + 4y' + 4.25y = 0$ with $y(0) = 1$ and $y'(0) = -2$ (10%)

(b) $y''' - y'' - 4y' + 4y = 6e^{-t}$ with $y(0) = 2$, $y'(0) = 3$ and $y''(0) = -1$ (15%)

3. The Laplace transform of a function $f(t)$ with $t \geq 0$ is defined as (25 分)

$$F(s) = \int_0^\infty e^{-st} f(t) dt$$

- (a) Find the Laplace transforms of $\sin(2t)$ and $\cos(2t)$, respectively. (10%)

- (b) Solve the initial value problem using the **Laplace transform** method: (15%)

$$y'' - 4y' + 3y = 2t - \frac{8}{3} \quad \text{with } y(0) = 0 \quad \text{and } y'(0) = -\frac{16}{3}$$

4. Solve the linear system
$$\begin{cases} x_1 + 2x_2 - 3x_3 = -11 \\ 10x_1 + x_2 + x_3 = 8 \\ 5x_2 + x_3 = 1 \end{cases} \quad (10 \text{ 分})$$

5. Find the Fourier series of the following periodic function (15 分)

$$f(x) = \begin{cases} -3 & \text{if } -\pi \leq x < 0 \\ 3 & \text{if } 0 \leq x < \pi \end{cases} \quad \text{and } f(x+2\pi) = f(x)$$