

國立成功大學

111學年度碩士班招生考試試題

編 號： 106

系 所： 水利及海洋工程學系

科 目： 工程數學

日 期： 0220

節 次： 第 2 節

備 註： 不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (20%)(a) Solve the initial-value problem

$$\frac{d^2x}{dt^2} + \omega^2x = \sin^2 \gamma t, x(0) = 0, x'(0) = 0. \quad (10\%)$$
 (b) Discuss under what frequency of γ the system is in pure resonance. (10%)
2. (10%) The inverse of a 2 by 2 matrix seems to have determinant = 1:

$$\det(\mathbf{A}^{-1}) = \det\left(\frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\right) = \frac{ad-bc}{ad-bc} = 1.$$
 What is wrong with this calculation? What is the correct $\det(\mathbf{A}^{-1})$?
3. (20%) Expand $f(x) = |x|$, $-\pi < x < \pi$
 - (a) in a Fourier series. (10%)
 - (b) Using the result obtained from (a), show that $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$ (10%)
4. (20%) (a) Find all values of x for which the matrix $\mathbf{A} = \begin{bmatrix} 1 & x \\ 1 & 1 \end{bmatrix}$ has an eigenvalue equal to 2. (5%)
 (b) Diagonalize the matrix $\mathbf{B} = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$. (5%)
 (c) Calculate the power \mathbf{B}^{2022} of the matrix $\mathbf{B} = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$. (5%)
 (d) Let \mathbf{Q} be any matrix which is symmetric and orthogonal. Find \mathbf{Q}^{2022} . (5%)
5. (30%) The faces of the thin rectangle plate in Fig. 1 with side $b = 1$, $a = 1$
 - (a) If both left and right sides are perfectly insulated, the upper side is kept at 0°C and the other sides are kept at 100°C , find the steady-state temperature $u(x, y)$ in the plate. (15%)
 - (b) If the lower side is kept at U_0 $^\circ\text{C}$, the upper side at U_1 $^\circ\text{C}$, and the other sides are kept at 0°C , find the steady-state solution. (15%)
 Note that both (a) and (b) satisfy Laplace's equation.

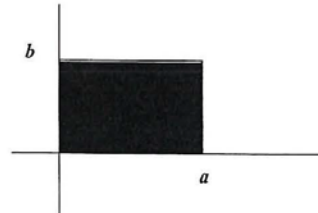


Figure 1: Square Plate