1. Solve the initial value problem (10%)

$$(x^2+y^2)\ dx-2xy\ dy=0,\quad \text{with } y(1)=2.$$

2. Solve the initial value problem (15%)

$$\begin{aligned} y_1'+y_2' &= 2\sinh t \\ y_2'+y_3' &= e^t \\ y_1'+y_3' &= 2e^t+e^{-t} \end{aligned} \qquad \text{with } y_1(0)=y_2(0)=1 \ \text{and}\ y_3(0)=0,$$

3. Find the unit tangent vector and the unit normal vector to the curve C: (15%)
$\vec{r}(t)=t^2\,\vec{i}+3t\,\vec{j}-t^3\,\vec{k}$ at the point P: (4,6,-8).

4. Using polar coordinates, evaluate $\iint_R \exp(-x^2-y^2)\ dx\ dy$ (15%)
where R is the annulus bounded by $x^2+y^2=1$ and $x^2+y^2=4$.

5. Evaluate the line integral $\int_C 3x^2\ dx+2yz\ dy+y^2\ dz$ (10%)
where the curve C: $\vec{r}(t)=t^2\,\vec{i}+(1-2t)\,\vec{j}+(2+5t)\,\vec{k}$ from the initial point A: (0,1,2) to the terminal point B: (1,-1,7).

6. Represent the function $f(x)=\cos\dfrac{\pi x}{L}$ where $0<x<L$ (10%)
by a Fourier half-range sine expansion and graph the corresponding periodic extension of $f(x)$ for a number of periods.

7. Solve the boundary-value problem in polar coordinates (15%)

PDE : $\dfrac{\partial^2 u}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial u}{\partial r}+\dfrac{1}{r^2}\dfrac{\partial^2 u}{\partial \theta^2}=0 \qquad 0<r<1$

BC : $u(1,\theta)=1+\sin\theta+\dfrac{1}{2}\sin 3\theta \qquad 0\le\theta<2\pi$

8. Evaluate the integral $\dfrac{1}{2\pi i}\oint_C \dfrac{e^{zt}}{z^2(z^2+2z+2)}\,dz$ (10%)
where $i=\sqrt{-1}$, $z=x+iy$, and C is the circle $|z|=3$ (counterclockwise).