1. Evaluate

$$\oint_c [Z^2 + 2Z^5 + \mathrm{Im}(Z)]dz$$

where Im is the imaginary part of a complex variable, c is a rectangular and its top points are 0, -2i, 2-2i and 2 as shown in Fig. 1.(10%)

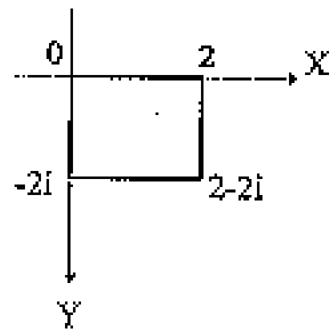

Fig.1

2. (1) What is the purpose of the Laplace transformation method? What are its advantages over the classical method.(8%)

(2) Determine the response of the damped mass system (Fig. 2) governed by

$$my'' + cy' + ky = r(t)$$

with the initial boundary conditions $y(0) = 0$ and $y'(0) = 0$, where m is the mass of the body, c the damping constant, k the spring modulus, and $r(t) = \delta(t-a)$ the Dirac delta function.(10%)

3. A tetrahedron is determined by three edge vectors $a = [2,0,3]$, $\bar{b} = [0,6,2]$, $\bar{c} = [3,3,0]$ as indicated in Fig. 3. Find the volume of the tetrahedron.(10%)

4. (1) The Laplace function

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

is considered on a rectangular boundary R. Draw a figure indicating what the possible boundary conditions are needed to solve this equation.(8%)

(2) Solve the wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

by the method of separating variables with initial conditions: $u(x,0) = f(x)$, $u_t(x,0) = g(x)$ and boundary conditions $u(0,t) = 0$, $u(L,t) = 0$.(10%)

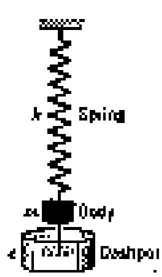

Fig. 2

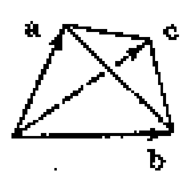

Fig. 3

(背面仍有題目,請繼續作答)

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5. $z = \tan^{-1}\frac{y}{x}$ is a curve in the space.

(1) find the length of the curve between point $(1,0,0)$ and $(1,0,2\pi)$.(8%)

(2) find the unit tangent and unit normal vectors of the curve at point $(0,1,\pi/2)$.(8%)

6. Solve the Euler equation

$$x^2\frac{d^2V}{dx^2}+\frac{5}{2}x\frac{dV}{dx}-\frac{V}{p}=\begin{cases}-\dfrac{x}{p} & ;x<1\\ 0 & ;x>1\end{cases}$$

with the boundary conditions: $V(0)=0$; $V(1^-)=V(1^+)$; $\frac{dV(1^-)}{dx}=\frac{dV(1^+)}{dx}$; $V(\infty)=0$.

where p is a constant, V=V(x) is a velocity distribution normal to a shoreline.(14%)

7. As shown in Fig.4, an artificial bar is periodic over intervals of width L and consists of a function $\delta(x)$ given by

$$\delta(x)=\begin{cases}D\cos\dfrac{\pi}{b_L}(x-NL); & NL-\dfrac{b_L}{2}\le x\le NL+\dfrac{b_L}{2}\\ 0 & \text{otherwise}\end{cases}$$

where $N=0,\ldots\ldots\ldots,N_b-1$, N_b is the number of bars, b_L the footprint of the bar on the bottom, and D is the bar height. The bar field can be represented by Fourier series,

$$\delta(x)=\sum_{n=0}^{\infty}D_n\cos(n\lambda x);\quad \lambda=\frac{2\pi}{L}$$

determine the coefficient D_n.(14%)

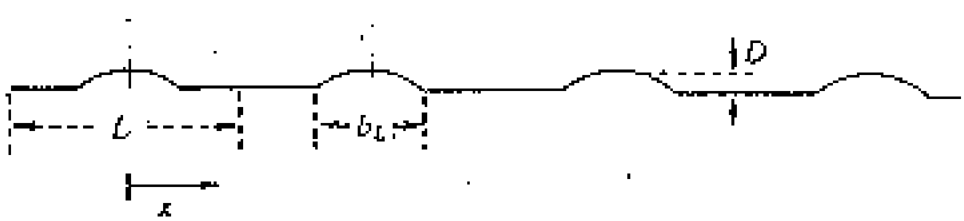

Fig. 4