1. (12%) Solve the initial value problem

$$y''' - y'' - y' + y = 0$$

$$y(0) = 2,\ y'(0) = 1,\ y''(0) = 0.$$

2. (12%) Solve the linear differential equation

$$y' - y = e^{2x},\quad y(0) = 2$$

3. (12%) Use the Laplace transform to solve the partial differential equation for a nonperiodic function $U_r(y,t)$

$$\frac{\partial U_r}{\partial t} = \nu \frac{\partial^2 U_r}{\partial y^2}$$

The initial and boundary conditions for U_r are

$$U_r(y,0) = 0,\quad U_r(0,t) = -U_B(t),\quad U_r(\infty,t) = 0.$$

(Hint: (i) use convolution theorem
& (ii) $L^{-1}\left[e^{-\sqrt{\frac{s}{\nu}}y}\right] = L^{-1}\left[e^{-\frac{y}{\sqrt{\nu}}\sqrt{s}}\right] = \frac{y}{2\sqrt{\nu\pi}}\, t^{-\frac{3}{2}} e^{-\frac{y^2}{4\nu t}}$)

4. (12%) Find out what type of conic section the following quadratic form represents and transform it to principal axis

$$Q = 17X_1^2 - 30X_1X_2 + 17X_2^2 = 128$$

5. (14%) Evaluate the surface integral $\iint \underline{F}\cdot\underline{n}\, dA$,
where $\underline{F} = [\,x^2, e^y, 1\,]$, S: $x+y+z=1$, $x \geqq 0$, $y \geqq 0$, $z \geqq 0$,
and $\underline{n}$ = unit normal vector of S.

6. (14%) Solve the boundary value problem by the method of separation of variables

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0,\quad 0 < x < a,\quad 0 < y < b$$

$$T(0,y) = 0,\quad \frac{\partial T}{\partial x}(a,y) = 0,\quad T(x,0) = T_0,\quad \frac{\partial T}{\partial y}(x,b) = 0$$

7. (12%) Evaluation of an improper integral by means of residues

$$\int_0^\infty \frac{dx}{1+x^4}$$

8. (12%) Find a linear fractional transformation that maps $|z| \leqq 1$ onto $|w| \leqq 1$ such that $z = \frac{i}{2}$ is mapped onto $w = 0$ and sketch the images of the lines $x = \text{const}$ and $y = \text{const}$.