1. (15%)

Solve the initial value problem (Show the details of your work).

$$y''(x) + y'(x) - 6y(x) = 0, \quad y(0) = 10, \quad y'(0) = 0$$

2. (15%)

Solve the initial value problem using Laplace transform

$$y''(t) + 2y'(t) + y = e^{t}, \quad y(0) = -1, \quad y'(0) = 1$$

3. (15%)

Evaluate the surface integral $\iint \underline{F} \cdot \underline{n}\, dA$, where $\underline{F} = (y^3, x^3, z^3)$,

Surface S: $x^2 + 4y^2 = 1, \quad x \geq 0, \quad y \geq 0, \quad 0 \leq z \leq h$

4. (15%)

Prove that eigenvectors of a symmetric matrix corresponding to different eigenvalues are orthogonal. Give an example.

5. (20%)

Find the solution of the wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

corresponding to the triangular initial deflection

$$f(x) = \begin{cases} \frac{2k}{L} x & \text{if } 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L - x) & \text{if } \frac{L}{2} < x < L \end{cases}$$

and initial velocity zero.

6. (20%)

Evaluate the improper integral by means of residues (Show the details of your work)

$$\int_{-\infty}^{+\infty} \frac{\sin 2x}{x^2 + x + 1}\, dx$$