

1. A body having mass one kilogram falls from rest toward the earth from a great height. As it falls, air resistance acts upon it, and we shall assume that this resistance is numerically equal to two times of the body's falling velocity. For simplicity, we may assume that the gravity acceleration equals  $10 \text{ m/s}^2$ . You are asked to:
- (a). Develop an equation for the body movement, based on the Newton's second law. (10%)
- (b). Find the velocity and distance fallen at time  $t$  seconds. (10%)

2. Evaluate the integral by using the Laplace transform or an appropriate method. (10%)

$$\int_0^{\infty} t e^{-2t} dt$$

3. Evaluate the integral by using the residue theory or an appropriate method. (10%)

$$\int_0^{\pi} \frac{1}{5 - 3 \cos \theta} d\theta$$

4. Solve the second-order differential equation. (20%)

$$y'' - 6y' + 25y = 0 \quad \text{with } y(0) = -3, \text{ and } y'(0) = -1.$$

5. Find the Fourier series of the periodic function. (20%)

$$f(x) = \begin{cases} 0 & \text{if } -2 < x < -1 \\ 1 & \text{if } -1 < x < 1 \\ 0 & \text{if } 1 < x < 2 \end{cases} \quad \text{with } f(x+4) = f(x)$$

6. Solve the system by using the Laplace transform or an appropriate method. (20%)

$$\begin{cases} y_1'' + 5y_1 - 2y_2 = 0 \\ y_2'' - 2y_1 + 2y_2 = 0 \end{cases} \quad \text{with } \begin{cases} y_1(0) = -1, & y_1'(0) = 0 \\ y_2(0) = 2, & y_2'(0) = 0 \end{cases}$$