每題 10 分共 10 題，必須列出計算過程

1. Solve the initial value problem

$y'' - 4y' + 4y = 0 \quad y(0) = 3 \,;\, y'(0) = 1$

2. Find the Laplace transform of $f(t)$

$$f(t) = \begin{cases} 2t & if\ 0 < t < 2 \\ 0 & if\ 2 < t < \pi \\ -\cos t & if\ t < \pi \end{cases}$$

3. Transform the following quardratic form to principle axes.

$Q = 5x_1^2 + 24x_1x_2 - 5x_2 = 0$

4. $\omega = f(x, y),\ x = r\cos\theta,\ y = r\sin\theta,$ Prove the following identity.

$\omega_{rr} + \dfrac{1}{r}\omega_r + \dfrac{1}{r^2}\omega_{\theta\theta} = \omega_{xx} + \omega_{yy}$

5. Evaluate the integral

$\displaystyle\int_{(0,2,1)}^{(2,0,1)} ze^x dx + 2yz\,dy + (e^x + y^2)dz$

6. Find the Frurier transform of the following function.

$$r(t) = \begin{cases} t + \dfrac{\pi}{2}, & -\pi < t < 0 \\ -t + \dfrac{\pi}{2}, & 0 < t < \pi \end{cases}, \quad r(t + 2\pi) = r(t)$$

7. Evaluate the integral

$\displaystyle\oint_c \dfrac{\tan z}{z^2 - 1}dz \quad c:|z| = 2\ (counterclockwise)$

(背面仍有題目，請繼續作答)

8. Consider a square vibrating membrane for which a=b=1 as shown in Fig.1.The eigenvalues are obtained by

$$\lambda_{mn} = c\pi\sqrt{m^2 + n^2} \qquad (1)$$

where $c^2 = T/\rho$, $T =$ the tension force, $\rho =$ the mass of the membrane per unit area. The eigenfunctions of the vibrating membrane is given by

$$u_{mn}(x,y,t) = (B_{mn}\cos\lambda_{mn}t + B_{mn}^*\sin\lambda_{mn}t)F_{mn}(x,y) \qquad (2)$$

in which

$$F_{mn}(x,y) = \sin\frac{m\pi x}{a}\sin\frac{n\pi x}{b} \qquad (3)$$

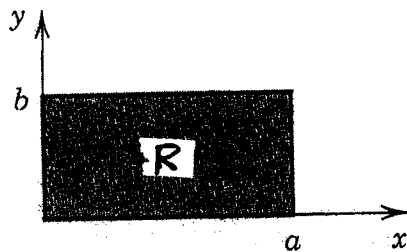Plot the nodal lines of the solutions $u_{11}, u_{12}, u_{21}, u_{22}, u_{13}$ and u_{31}.

Fig. 1　　　Rectangular membrane

9. Find the directional derivative of $f(x,y,z) = 2x^2 + 3y^2 + z^2$ at the point $p:(2,1,3)$ in the direction of the vector $\vec{a} = \vec{i} - 2\vec{k}$.

10. Find the radius of convergence and center of the power series

$$\sum_{n=0}^{\infty}\frac{(2n)!}{(n!)^2}(z-3i)^n$$