本試題是否可以使用計算機： ☑可使用 ， □不可使用 （請命題老師勾選）

每題十分，共有十題，必須寫出計算過程．

1. Solve

$$(\cos x \sin x - xy^2)dx + y(1-x^2)dy = 0$$

subject to $y(0)=2$ (10%)

2. Find the Laplace transform of the function shown in Fig.1 . (10%)

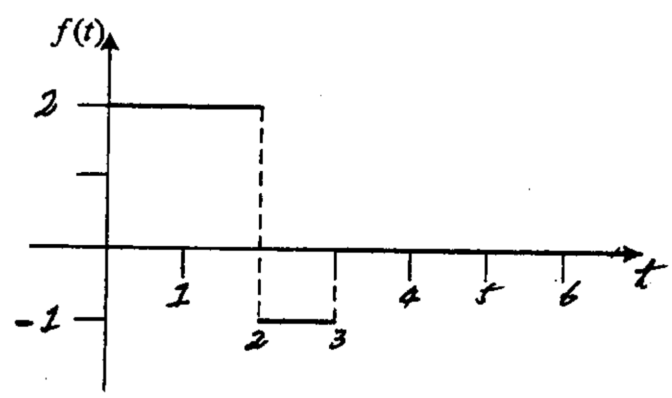


Fig.1

3. The general Legendre polynomial can be written as

$$P_n(x) = \sum_{k=0}^{n/2} \frac{(-1)^k (2n-2k)!}{2^n k!(n-k)!(n-2k)!} x^{n-2k}$$

where $[n/2]$ is the greastest integer not greater than $n/2$. Use the explicit Legendre polynomials $P_2(x)$ and $P_3(x)$ to evaluate $\int_{-1}^{1} P_n(x)P_m(x)dx$ for $n=2$ and $m=3$. (10%)

4. Find an equation of the plane containing $(1,7,-1)$ that is perpendicular to the line of intersection of $-x+y-8z=4$ and $3x-y+2z=0$. (10%)

(背面仍有題目,請繼續作答)

5. Let A be an 3×3 matrix. The matrix that is the transpose of the matrix of cofactors corresponding to the entrie of A :

$$\begin{bmatrix} c_{11} & c_{21} & c_{31} \\ c_{12} & c_{22} & c_{32} \\ c_{13} & c_{23} & c_{33} \end{bmatrix}$$

is called the adjoint of A and is denoted by $\mathrm{Ad_j}\,A$. Prove the inverse of A^{-1} expressed by

$$A^{-1} = \frac{1}{\det A}\ \mathrm{Ad_j}\,A \qquad (10\%)$$

6. Find the directional derivative of $f(x,y,z)=xy^2-4x^2y+z^2$ at $(1,-1,2)$ in the direction of $6\vec{i}+2\vec{j}+3\vec{k}$. (10%)

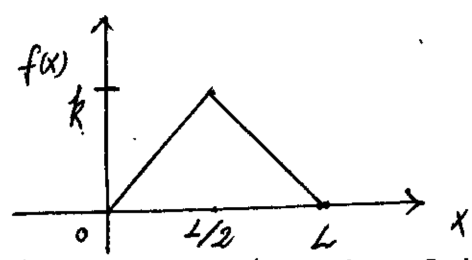
7. Evaluate

$$\int_c (y+yz)dx+(x+3z^3+xz)dy+(9yz^2+xy-1)dz$$

in which c represents a path between $(1,1,1)$ and $(2,1,4)$. (10%)

8. The $1-D$ wave equation is solved by

$$u(x,t)=\frac{1}{2}[f(x+ct)+f(x-ct)]$$



Plot the deflection of $u(x,0)$, $u(x,2L/5c)$, and $u(x,L/c)$, where L is the length of string and c is the phase velocity . (10%)

9. Let $f(z)=u(x,y)+iv(x,y)$ be defined and continuous in some neighborhood of a point $z=x+iy$ and differentiable at z itself . If $f(z)$ is analytic in a domain D, find the Cauchy-Rieman equations . (10%)

10. Evaluate the integral $\int_0^{2\pi}\frac{d\theta}{2+\cos\theta}$ (10%)