編號： 144　系所：水利及海洋工程學系乙組　　科目：工程數學

本試題是否可以使用計算機：☑可使用，☐不可使用　（請命題老師勾選）

1. (a) Find the area of the region bounded by the x-axis, the line x=2, and the graph of

$$f(x) = x^3 - 2x^2 + 3$$ (10%)

(b) Evaluate $\int_0^{\pi} x^2 \sin x \, dx$ (10%)

2. Solve the Bernoulli equation (Hint: Let a new variable $u = y^{-2}$). (20%)

$$\frac{dy}{dt} + y = y^3 \qquad \text{with } y(0) = \frac{1}{\sqrt{3}}$$

3. (a) Solve the complex quadratic equation of Z, where $Z = a + bi$. (10%)

$$Z^2 - (5+i)Z + (8+i) = 0$$

(b) Solve the complex equation $Z^8 = 1$, and plot all solutions in the complex plane. (10%)

4. Solve the second-order differential equation. (20%)

$$y'' + y = 2t \qquad \text{with } y(\pi/4) = \pi/2, \text{ and } y'(\pi/4) = 2 - \sqrt{2}.$$

5. Solve the Laplace equation of $U(x, y)$ (20%)

$$PDE: \quad \nabla^2 U(x, y) = 0 \qquad for \quad x^2 + y^2 < 16$$
$$BC: \quad U(x, y) = x^2 \qquad for \quad x^2 + y^2 = 16$$