編號： 141　系所：水利及海洋工程學系甲組　　科目：工程數學

本試題是否可以使用計算機：☑可使用，☐不可使用　（請命題老師勾選）

1. (a) Find the area A of the region bounded by the graphs of $f(x)=\sqrt{x}$ and $g(x)=-x$ on the interval of x: [1, 4], i.e., (10%)

$$A=\int_1^4 [f(x)-g(x)]dx$$

(b) Find the area of the region bounded by the graphs of $f(x)=\sqrt{x}$ and $g(x)=x^2$. (10%)

2. An object having mass ***m*** falls from rest toward the earth from a great height under the gravity force with gravity acceleration ***g***. As it falls, air resistance acts upon it, and this resistance F_D is proportional to the square of the velocity V, i.e., $F_D=\alpha V^2$. According to the Newton's second law, the equation of motion for the falling object can be written as (20%)

$$m\frac{dV}{dt}=mg-\alpha V^2 \qquad \text{with } V(0)=0$$

Please find the falling velocity $V(t)$ and the terminal velocity V_∞ that is the velocity at the time approaching infinite.

3. Solve the linear system (20%)

$$\begin{aligned} x_1+x_2+x_3+x_4&=0\\ 2x_1-2x_2-x_3+2x_4&=0\\ 3x_1-x_2+2x_3+2x_4&=0\\ x_1-x_2-2x_3+x_4&=0 \end{aligned}$$

4. Solve the second-order differential equation of the underdamped forced motion. (20%)

$$y''+2y'+2y=2\sqrt{2}\cos(\sqrt{2}\,t) \qquad \text{with } y(0)=0, \text{ and } y'(0)=0.$$

5. Solve the Laplace equation of $U(r,\theta)$ in polar coordinates. (20%)

$$PDE:\ \frac{\partial^2 U}{\partial r^2}+\frac{1}{r}\frac{\partial U}{\partial r}+\frac{1}{r^2}\frac{\partial^2 U}{\partial \theta^2}=0 \qquad \text{for } \begin{matrix}0<r<1\\ 0<\theta\le 2\pi\end{matrix}$$

$$BC:\ U(1,\theta)=2+\sin\theta+3\sin 3\theta$$