※ 考生請注意：本試題 □可 ☑不可 使用計算機

1. (20%)

Solve the following initial value problems by the Laplace transform.

$y''+k^2y=r(t), y(0)=0, y'(0)=\frac{1}{k}$, k is a positive constant, where

(1) (10%) $r(t)=\cos wt$, where w is a positive constant.

(2) (10%) $r(t)=\begin{cases}1, 0<t<1\\0, t>1\end{cases}$.

2. (20%)

Given a matrix $\mathbf{A}=\begin{bmatrix}1-k & 1 & 0\\ 1 & 1-k & 0\\ 0 & 0 & 1\end{bmatrix}$, where k is a constant to be determined.

(1) (10%) Determine the relationship between the rank of **A** and the values of k.

(2) (10%) Determine the value of k if the matrix **A** is an orthogonal matrix.

3. (20%)

Given the line integral $\int_{(1,\frac{\pi}{4},2)}^{(2,\frac{\pi}{2},4)} kxyz^2dx+\left(x^2z^2+z\cos yz\right)dy+\left(kx^2yz+y\cos yz\right)dz$, where k is a constant to be determined.

(1) (10%) Determine the value of k that the line integral is independent of path.

(2) (10%) Evaluate the path independent integral for the derived k.

4. (20%)

(1) (10%) Find the Fourier integral representation of the function $f(x)=\begin{cases}1, |x|<1\\0, |x|>1\end{cases}$.

(2) (10%) Using the obtained result of (1) to show that $\int_0^\infty \frac{\sin w}{w}dw=\frac{\pi}{2}$.

5. (20%)

Solve the equation $\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}=0, u(x,0)=0, u(x,b)=f(x), \left.\frac{\partial u}{\partial x}\right|_{x=0}=0, \left.\frac{\partial u}{\partial y}\right|_{x=a}=0.$