

系所組別 水利及海洋工程學系甲、乙組

考試科目 工程數學

考試日期：0307，節次：3

※ 考生請注意：本試題 可 不可 使用計算機

1. (30%) Consider line integrals $\Gamma = \int_C \mathbf{F} \cdot d\mathbf{r} = \int_C F_1 dx + F_2 dy$, where $\mathbf{F} = (F_1, F_2) = \left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right)$, $\mathbf{r} = (x, y)$ is a position vector, and $C: x^2 + y^2 = 1$ oriented counter-clockwise.

(a) Calculate Γ directly.

(b) State the Stokes theorem for transformation between surface and line integrals.

(c) Can you apply Stokes theorem to obtain the results from (a)? State clearly the reason why?

Note: The expression " \cdot " represents the dot product.

2. (15%) Evaluate $\oint_C \cot z dz$, where the contour C is the circle $|z| = 4$.

3. (25%) The Dirichlet problem for the exterior of a circle (radius= a) that satisfies the Laplace equation is

$$\nabla^2 u = u_{xx} + u_{yy} = u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0 \quad \text{for } x^2 + y^2 > a^2$$

$$\text{BC's: } u(a, \theta) = h(\theta)$$

$$u \text{ is bounded as } x^2 + y^2 \rightarrow \infty$$

(a) Solve this problem using separation of variables.

(b) The series obtained from (a) can be summed explicitly, which is known as Poisson's formula. Find this formula.

4. (30%) Determine whether each statement is true or false. If a statement is true, give a reason. If a statement is false, provide an example that shows that the statement is not true in all cases or cite an appropriate statement. Note in the following the \mathbf{x} and \mathbf{b} are column vectors and "det" means "determinant"

(a) For any matrix \mathbf{A} , the matrix $\mathbf{A}\mathbf{A}^T$ is symmetric.(b) If the matrices \mathbf{A} , \mathbf{B} , and \mathbf{C} satisfy $\mathbf{A}\mathbf{B}=\mathbf{A}\mathbf{C}$, then $\mathbf{B}=\mathbf{C}$.(c) If \mathbf{A} can be row reduced to the identity matrix, then \mathbf{A} is nonsingular.(d) If \mathbf{A} is a square matrix, then the system of linear equations $\mathbf{A}\mathbf{x}=\mathbf{b}$ has a unique solution.(e) The determinant of a square matrix \mathbf{A} is a nonzero scalar.(f) If \mathbf{A} is a 3×3 matrix with $\det(\mathbf{A}) = 5$, then $\det(2\mathbf{A}) = 10$.(g) If \mathbf{A} and \mathbf{B} are nonsingular $n \times n$ matrices, then $\mathbf{A} + \mathbf{B}$ is a nonsingular matrix.(h) If the determinant of an $n \times n$ matrix \mathbf{A} is nonzero, then $\mathbf{A}\mathbf{x} = 0$ has only the trivial solution.(i) An invertible square matrix \mathbf{A} is called orthogonal if $\mathbf{A}^{-1} = \mathbf{A}^T$. Then $\det(\mathbf{A}) = \pm 1$.(j) If \mathbf{x} is the eigenvector of $\mathbf{A}\mathbf{x} = \lambda\mathbf{x}$ with λ being eigenvalue, then the determinant of $\mathbf{A} - \lambda\mathbf{I}$ is zero.