系所組別： 航空太空工程學系甲乙丙丁組

考試科目： 工程數學

考試日期：0219，節次：3

※ 考生請注意：本試題 ☑可 ☐不可 使用計算機

1. Solve the following equations.

10%(a). $y'(x)=\dfrac{3x^2-y}{x-1}$, $y(0)=0$, $y(x)=?$

10%(b). $y''-2y'+2y=\sin x$, $y(0)=-\dfrac{2}{5}$, $y'(0)=-\dfrac{1}{5}$, $y(x)=?$

2.

10%(a). Is the following set of vectors linearly independent or dependent.
(Show the details of your work.)

$$v_1=[3 \quad 0 \quad 2 \quad 2]^T,\ v_2=[-6 \quad 42 \quad 24 \quad 54]^T,\quad v_3=[21 \quad -21 \quad 0 \quad -15]^T$$

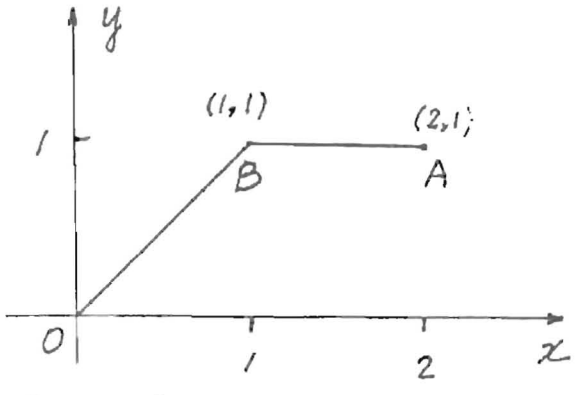
10%(b). Suppose that $f(x^*,y^*)$ is the local minimum of the function

$$f(x,y)=x^2-4xy+y^3,$$

what is the local minimizer (x^*,y^*) ?

3. (a). Given a trajectory represented by $\vec{r}(t)$, determine the velocity $\vec{V}(t)$ and the acceleration $\vec{a}(t)$ by using the vector differentiation. Express them with $\dot{s}$ (the speed along the tangent direction), $\ddot{s}$ (the acceleration along the tangent direction), ρ (the radius of curvature), $\vec{e}_t$ and $\vec{e}_n$ (the unit vectors along the tangential and normal directions, respectively). (10%)

(b). Determine the integral, $\int_{\vec{r}_o}^{\vec{r}}\vec{V}\cdot d\vec{r}$, where $\vec{V}=3y\vec{i}-2x\vec{j}$, and $\vec{r}$ represents the line beginning from point O, passing through point B, and ending at point A as shown in the following figure. $\vec{i}$ and $\vec{j}$ are the unit vectors along x- and y- axes, respectively. (10%)



(背面仍有題目,請繼續作答)

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4. (a) (6%)

Classify the following partial differential equation (PDE) to be elliptic, parabolic, or hyperbolic PDE:

$$2u_{xx}+10u_{xy}+8u_{yy}+xu_x-yu_y=0$$

(b) (14%)

Classify the Sturm-Liouville problem as regular, periodic, or singular; find the eigenvalues and corresponding eigenfunctions.

i) $y''+\lambda y=0; \qquad y(0)=0, y'(4)=0$

ii) $y''+\lambda y=0; \qquad y(0)=y(4), y'(0)=y'(4)$

5. Consider the function of complex variable

$$f(z)=\frac{1}{z^2(1-e^z)}$$

(a). Identify all singular points for $f(z)$. (6%)

(b). Evaluate the following integral using the *Residue Theorem*

$$\oint_C f(z)dz$$

where C is the circle $|z|=1$. (10%)

(c). If C is the circle $|z|=6$, will the result in (b) change? (4%)