1. (20 %)
   a). Let
   $$f(z) = \frac{z}{(4+z^2)(2+z)},$$
   find the singular points and the corresponding residues.
   b). Compute
   $$\int_{-\infty}^{\infty} \frac{x}{(4+x^2)(2+x)} dx.$$

2. Consider an elliptic surface
   $$x^2 + \frac{y^2}{4} + \frac{z^2}{4} = 1.$$
   a). Find the point on the surface at which the normal line of the surface going through (2,3,3). (10 %)
   b). Write the equation representing the 2-D plane determined by z-axis and the normal line in a). (10 %)

3. Solve the following equations (20%)
   a). $(2x+y)dx + (x+2y)dy = 0, y(0) = 0;$
   b). $y'''(x) - 3y''(x) + 3y'(x) - y(x) = 6e^x, y(0) = 0, y'(0) = 0, y''(0) = 2.$

4. (20 %)
   a). Prove that the eigenvalues of a real symmetric matrix are real. (10 %)
   b). Prove that the product of two unitary matrices is unitary. (5 %)
   c). Prove that the main diagonal elements of a hermitian matrix must be real. (5 %)

5. (20 %)
   a). Find the Fourier series representation of
   $$f(x) = \frac{x^2}{2}, \qquad -\pi < x < \pi.$$
   b). Find the sum of the following series
   $$1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} - \cdots$$