※ 考生請注意：本試題不可使用計算機

1. A gas-filled pneumatic strut in an automobile suspension system behaves like a piston-cylinder apparatus. At one instant when the piston is $L = 0.15m$ away from the closed end of the cylinder, the gas density is uniform at $\rho = 18kg/m^3$ and the piston begins to move away from the closed end at $V = 12m/s$. Assume as a simple model that the gas velocity is one-dimensional and proportional to distance from the closed end; it varies linearly from zero at the end to u=V at the piston. Find the rate of change of gas density at this instant. Obtain an expression for the average density as a function of time. (20%)

2. Consider the simplified momentum equation shown below for a steady, incompressible flow with constant viscosity and negligible body forces. The problem has a characteristic length scale *L* and velocity scale *U*.

$$\underbrace{\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)}_{A}=\underbrace{-\frac{\partial p}{\partial x}}_{B}+\underbrace{\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)}_{C}$$

Determine the proper non-dimensionalization of the pressure if (a) term $A \approx 0$
And if (b) term $C \approx 0$. Provide real world examples when this might be the case. (20%)

3.Answer the following questions: (20%)

(1) What conditions are necessary before you can use a stream function to solve for the flow field? (5%)
(2) What conditions are necessary before you can use a potential function to solve for the flow field? (5%)
(3) What conditions are necessary before you can apply Bernoulli's Equation to relate any two points in a flow field? (5%)
(4) Under what conditions does the circulation around a closed fluid line remain constant with respect to time? (5%)

(背面仍有題目,請繼續作答)

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4. Consider a steady velocity field $\vec{V} = (u, v) = (x^2 + y^2, -2xy)$ where (u, v) are the Cartesian velocity components and (x, y) is the position in Cartesian system. A triangular path C passing through (0,0), (1,0) and (1,2) is shown in the attached Figure A.

(a) Verify whether this velocity field is compressible or incompressible.

(b) Verify whether this velocity field is rotational or irrotational.

(c) Calculate the circulation $\Gamma = \oint_C \vec{V} \cdot d\vec{r}$ along path C in counter-clockwise direction as shown in Figure A.

(20%)

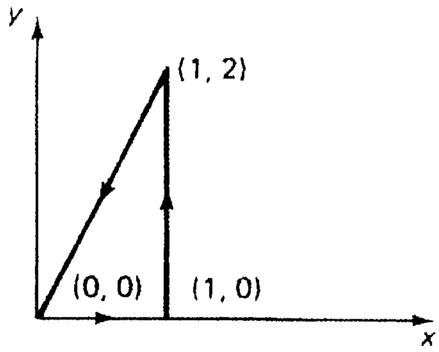

Figure A

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Problem 5 (20%)

Consider a vane cascade struck by a continuous jet of water that leaves a nozzle, which has an exit cross-sectional area A, with a jet speed V. The vanes move with a constant speed U. Note that all the mass flow leaving the jet crosses the vanes. The curvature of the vanes are described by angles Θ_1 and Θ_2, as shown. Evaluate the nozzle angle, α, required to ensure that the jet enters tangent to the leading edge of each vane (10%). Calculate the force that must be applied to maintain the vane speed constant (10%).