系所組別： 航空太空工程學系甲乙丙丁組

考試科目： 工程數學

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (20%) For equation $e^z = i$, considering $z = a + ib$ is a complex variable, where a and b are real numbers.

(a) Find all the solutions of a and b to the above equation, and

(b) Find the modulus and argument of z.

2. (20%) At a point P_1 located at (3,2,-5) in space, suppose we have another point P_2 at (4,-2,6). Determine the followings:

(a) the **unit** direction vector from P_1 to P_2 (5%);

(b) the gradient of $F(x,y,z) = y\,e^x\,\mathbf{i} + (x+y)z\,\mathbf{j} + z^2x\,\mathbf{k}$ (i.e. $\nabla F(x,y,z)$), where ($\mathbf{i}, \mathbf{j}, \mathbf{k}$) is the unit basis vector of the Cartesian coordinate system (5%);

(c) the gradient of F at P_1 (5%);

(d) direction derivatives of the function $F(x,y,z)$ at P_1 in the direction of P_2 (5%).

(背面仍有題目,請繼續作答)

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3. (20%) Consider a data set listed below:

(0, 6), (2, 8), (6, 9), (10, 12)

(a) Find the linear function, $y=ax+b$, by least square method to approximate the data relation, x as input and y as output. (13%)

(b) Give a formula or an equation which could be used to estimate the average error in percentage for the function in a. (7%)

4. (20%) Define the Fourier transform of a function $f(x)$ as

$$\mathbf{F}[f(x)] \equiv F(\omega) \triangleq \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx.$$

(a) Given $f(x) = \begin{cases} e^{-ax}, & x \geq 0 \\ 0, & x < 0 \end{cases}$ $(a > 0)$, find its Fourier transform $F(\omega)$.

(b) Define the convolution integral of two functions $f(x)$ and $g(x)$ as

$$[f * g](x) \triangleq \int_{-\infty}^{\infty} f(x-\xi) g(\xi) d\xi.$$

Prove the *Fourier convolution theorem*:

$$\mathbf{F}[f * g] \equiv F(\omega) G(\omega)$$

5. 20%)

a) Find the radius of convergence and interval of convergence:

i) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n} (x-2)^n$ **ii)** $\sum_{n=1}^{\infty} 2^n (x-1)^n$

b) Using the method of power series, find the recurrence relation and two linearly independent solutions for the following ODE:

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + xy = 0$$