

※考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (25%) Velocity field, $\vec{V} = (Ax, -2Ax)$, $A = 2$:

- (a) Equation of the streamlines in the xy plane. (5%)
- (b) Streamline plot through point (4, 8). (5%)
- (c) Velocity of particle at point (4, 8). (5%)
- (d) Position at $t = 2$ of particle located at (4, 8) at $t = 0$. (5%)
- (e) Velocity of particle at position found in (d). (5%)

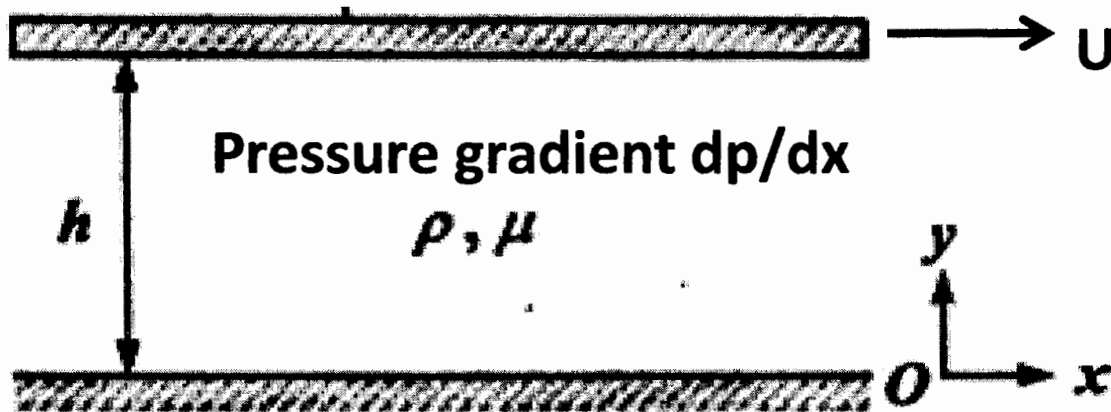
2. (25%) Consider flow over a small object in a viscous fluid, such as a small particle settling in a glass of water. Analysis of the Navier-Stokes equations shows that the inertial terms are much smaller than the viscous and pressure terms in this case. It turns out, then, that fluid density drops out of the Navier-Stokes equations. Such flows are called creeping flows. The only important parameters in creeping flow are the particle velocity U (relative to the fluid), the fluid viscosity μ , and the particle length scale d . For three-dimensional bodies, like spheres, creeping-flow analysis yields very good results. It is uncertain, however, if creeping flow applies to two-dimensional bodies, such as cylinders, since even though the diameter may be very small, the length of the cylinder is infinite.

- (a) Apply the Pi theorem to generate an expression for the two-dimensional drag force D_{2-D} as a function of the other parameters in the problem. Be careful: two-dimensional drag has dimensions of force per unit length, not simply force. (10%)
- (b) Is your result in part (a) physically plausible? If not, explain why not. (5%)
- (c) It turns out that fluid density ρ cannot be neglected in analysis of creeping flow over two-dimensional bodies. Repeat the dimensional analysis, this time including ρ included as a parameter. Find the resulting nondimensional relation between the parameters in this problem. (10%)

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3. (25%) As shown in the figure below, fluid with density ρ and viscosity μ is between two infinite parallel plates with pressure gradient dp/dx . Assume the spacing between two plates is h , the bottom wall is fixed and top wall is moving with constant speed U .

- Write down the simplified governing equation for this flow field with boundary conditions. (5%)
- Find the velocity distribution $u(y)$? (5%)
- Calculate the shear stress τ on the bottom wall. (5%)
- Calculate the volume flow rate Q per unit depth. (5%)
- Find y coordinate of the point that has maximum velocity? (5%)



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4. (25%) Consider a flow in a two-dimensional duct of a length L , whose walls are converging linearly in the streamwise direction, denoted as the x direction in the figure below. Assume the flow be inviscid and incompressible. The duct width at $x=0$ and L are h_0 and $h_0/2$, respectively; the streamwise velocity u at $x=0$ and L are U_0 and $2U_0$, respectively. Let the streamwise velocity u be a function of x only, $u=u(x)$. Find the expressions of the velocity in the y direction, called v , and the streamwise pressure gradient, called $\frac{\partial p}{\partial x}$, in terms of U_0 , L and ρ , where ρ denotes the density of the flow.

