

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (20%) Ball,  $m = 0.1kg$ , released from rest at  $y_0 = 200m$ .  $y$  is the height position of the ball.

Air resistance,  $F_D = kV^2$ , where  $k = 10^{-4} N \cdot s^2 / m^2$ ,  $V$  is the velocity of the ball, and the gravity,  $g = 9.8m/s^2$

- (a) Find the total force acting on the ball at each time,  $t$ .

- (b) Now, set the velocity,  $V$ , is also a function of  $y$  and  $\frac{dV}{dt} = \frac{dV}{dy} \frac{dy}{dt} = V \frac{dV}{dy}$

Find the ordinary equation for the velocity at each position,  $y$ , and solve the solution of the ODE.

- (c) Find the speed at which the ball hits the ground,  $y = 0m$ .

2. (20%) Consider the two-dimensional flow field defined by the following velocity components,  $(u, v)$ :

$$u = x, \quad v = -y$$

For this flow field find the equation of:

- (a) The streamline through the point (1,1) at  $t=0$
- (b) The pathline for a particle released at the point (1,1) at  $t=0$
- (c) The streakline at  $t=0$  which passes through the point (1,1).

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3. Consider the two-dimensional fluid flow with velocity components:

$$u = ax + bx^2 + cy ; v = dy + exy$$

where a, b, and c are constants greater than zero.

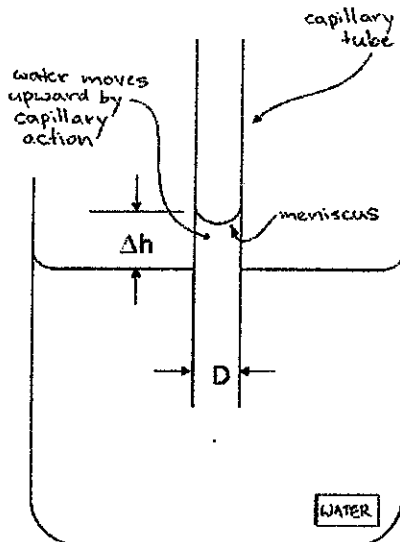
(a) Assume density is constant and the mass is conserved, what can be said about constant d and e. (7%)

(b) Determine if the vorticity is zero somewhere in the flow field. Is this flow an irrotational flow? (7%)

(c) Find the rate of angular deformation in xy plane of this flow field? (6%)

4. When a small tube is dipped into a pool of liquid, surface tension causes a meniscus to form at the free surface, which is elevated or depressed depending on the contact angle at the liquid-solid-gas interface. Experiments indicate that the magnitude this capillary effect  $\Delta h$ , is function of the tube diameter D, liquid specific weight  $\gamma$  and the surface tension  $\sigma$ . Determine the number of independent non-dimensional parameters by using dimensional analysis. (20%)

Hint: The dimension of surface tension  $\sigma$  is force per unit length.



5. Figure below shows a flow between a rotating inner cylinder (radius  $r=R_i$ ) with angular velocity  $\omega$  and stationary outer cylinder (radius  $r=R_o$ ) inside a rotational viscometer. Assume the flow can be modelled as Couette flow since the gap is so small and the density and viscosity of the fluid is  $\rho$  and  $\mu$ . Please answer the following questions:

(a) Derive the steady state velocity field  $U=f(r)$  between at  $r=R_o$  and  $r=R_i$ ? (10%)

(b) If the viscometer cylinder height is  $L$ , find out the total torque ( $T_{viscous}$ ) acting on the inner cylinder wall due to fluid viscosity. (5%)

(c) Derive the viscosity ( $\mu$ ) in terms of total torque ( $T_{viscous}$ ) acting on the inner cylinder, cylinder height ( $L$ ), angular velocity ( $\omega$ ), outer radius ( $R_o$ ) and inner radius ( $R_i$ ). (5%)

