編號：140、144、176　　國立成功大學 107 學年度碩士班招生考試試題

系　　所：航空太空工程學系、民航研究所、能源國際碩士學位學程

考試科目：工程數學　　考試日期：0205，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10%)

Find the rank, a basis for row space, and a basis for the column basis of the following matrix.

$$\begin{bmatrix} 1 & 2 & -1 & 3 & 1 \\ 0 & 1 & -3 & 2 & 3 \\ 2 & 3 & 1 & 4 & -1 \\ -1 & 2 & 2 & 2 & -5 \\ 3 & 1 & -1 & 2 & 4 \end{bmatrix}$$

2. (10%)

Prove (a) $\nabla \cdot (\nabla \times \vec{v}) = 0$ and (b) $\nabla \times (\nabla \varphi) = 0$ (φ is a scalr field).

3. (10%)

Find (a) a unit vector perpendicular to the plane $4x + 2y + 4z = 7$ and (b) the distance from the origin to this plane.

4. (20%)

Derive y(t) for the ordinary differential equation given in the following:

$\ddot{y} + 2\dot{y} + 2y = f(t)$, where $y(0) = 1$ and $\dot{y}(0) = 0$, and

$$f(t) = \begin{cases} 2t, & 0 \le t < 1, \\ 0 & 1 \le t < \infty. \end{cases}$$

5. (20%)

Given $F(x,y,z)=f(r)$, where $r=(x^2+y^2+z^2)^{1/2}$.

If $\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 F}{\partial z^2} = 0$, find $f(r)$.

6. (10%) (a) Find the eigenvalues of the matrix and (b) find eigenvectors corresponding to each eigenvalue.

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

7. (20%)

a) Find the singular points and the corresponding residues:

$$\frac{z+2}{z^3+4z}$$

b) Evaluate the integral:

$$\int_{-\infty}^{\infty} \frac{x+2}{x^3+4x} dx$$