

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

Q.1 Consider the double-pendulum system shown in Fig.1, where gravity is the only external force. Assume the displacement angles of the pendulums are small enough to ensure that the spring is always horizontal. The pendulum rods are taken to be massless, of length l , and the springs are attached $3/4$ of the way down.

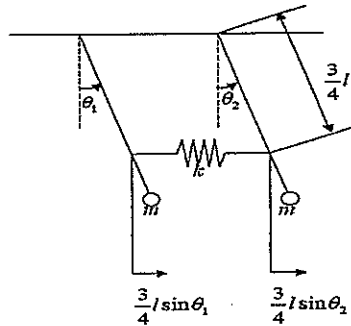


Fig.1 Double-pendulum system.

- (a). Please write the equations of motion. (10%)
- (b). Derive the linearized model. (5%)
- (c). For given non-zero initial angles, find the steady state values of θ_1 and θ_2 , respectively, if exist. (5%)

Q.2 Consider the system shown in Fig.2

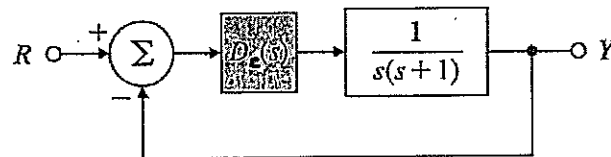


Fig. 2 A feedback control system.

where R and Y represent s-domain system reference and output, respectively.

- (a) Let $D_c(s) = K$, where $K > 0$. Please find the natural frequency and damping ratio. (5%)
- (b) Based on (a), for any given desired settling time and maximum overshoot, is it possible to meet them at the same time? Please provide your proof. (5%)

(c) Let $D_c(s) = K \frac{(s + \alpha)^2}{s^2 + \omega_0^2}$. If the system is stable, is it possible to track a sinusoidal reference input

$r = \sin(\omega_0 t)$ with zero steady-state error. Please provide detail proof. (10%)

- (d) Use Routh's criterion to find the range of K , α and ω_0 such that the closed-loop system is stable. (10%)

Q.3 For the system shown in Fig. 3,

- (a) draw the Nyquist plot for $K = 1$ (10%)
- (b) apply the Nyquist criterion to determine the range of values of K (positive and negative) for which the system will be stable, (5%) and
- (c) determine the number of roots in the RHP for those values of K for which the system is unstable based on the Nyquist plot. (5%)
- (d) draw the Bode plot for $K = 1$ and determine the gain margin and phase margin of the resulting closed-loop system. (10%)

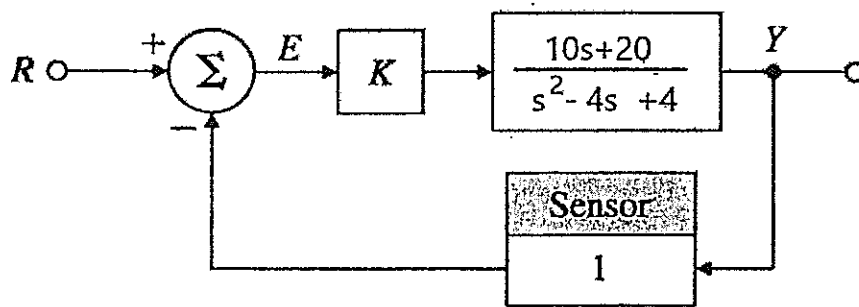


Fig. 3 System block diagram

Q.4 The block diagram of a positioning servomechanism is shown in Fig. 4.

- (a) For $K = 2$, draw the root locus with respect to K_T . (8%)
- (b) For $K = 2$, determine K_T so that damping ratio $\zeta = 0.707$ is achieved for the closed-loop poles. (6%)
- (c) For the values of K and K_T in part (b), what will be the system steady state error to a unit ramp input. (6%)

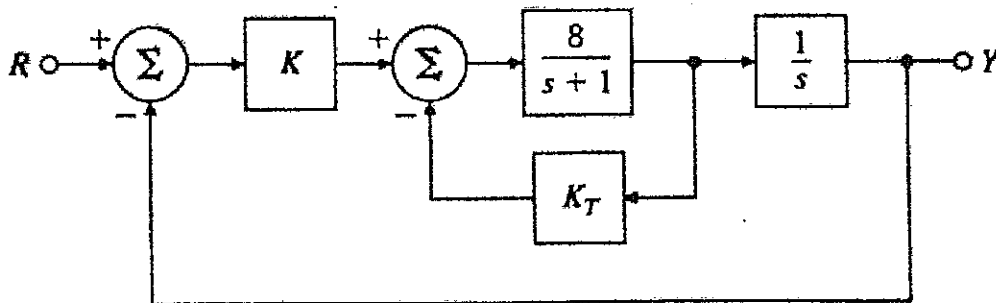


Fig. 4 A multi-loop control system