※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Consider the equation $y''+5y'+6y=u(t-1)-u(t-2)$, $y(0)=1$, and $y'(0)=0$,

   where $u(t-a)=\begin{cases}0 & 0\le t<a,\\ 1 & a\le t.\end{cases}$ Solve for $y(t)$ with Laplace transform. (20 points)

2. (a) Find the tangent to the ellipse $\frac{1}{4}x^2+y^2=1$ at $P:(\sqrt{2},\ 1/\sqrt{2})$ (6 points)

   (b) Using Green's theorem, evaluate the line integral $\oint_C \mathbf{F}(\mathbf{r})\bullet d\mathbf{r}$ counterclockwise around the boundary C of the region R

   where $\mathbf{F}=\left[x^2e^y,\ y^2e^x\right]$, C the rectangle with vertices (0, 0), (2, 0), (2, 3), (0, 3) (6 points)

   (c) $f=\frac{1}{r}$, $r=\sqrt{x^2+y^2+z^2}$, find $\iint_S \nabla f\cdot\hat{n}dA$ for S: $x^2+y^2+z^2=a^2$ (8 points)

(背面尚有題目)

3. (a) Find the singular points and the corresponding residues: (10 points)

(i) $\dfrac{1}{z(z-1)^2}$ (ii) $\cot(z)$

(b) Using the residue theorem, evaluate $I=\int_{-\infty}^{\infty}\dfrac{x+2}{x^3+4x}dx$ (10 points)

4. (a) Define the even and odd functions, respectively. (6 points)

(b) Is a Fourier cosine series an even or odd function?

How about a Fourier sine series? Show your arguments. (4 points)

(c) Show that any function, $f(x)$, can be uniquely decomposed into the sum of an even function and an odd function. (10 points)

5. Consider a two-dimensional Laplace's equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0 \quad \text{in the domain of } 0\le x\le a \text{ and } 0\le y\le b$$

with the boundary condition of
$$\begin{cases} u=y & \text{at } x=0,\ 0\le y\le b \\ u=c & \text{at } x=a,\ 0\le y\le b \\ u=c & \text{at } y=0,\ 0\le x\le a \\ \dfrac{\partial u}{\partial y}=0 & \text{at } y=b,\ 0\le x\le a \end{cases}$$

Find the solution of $u(x,y)$ (20 points)